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A benchmark Ecological Stock-Flow-Consistent Input-Output model for Denmark

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Abstract

This paper aims to develop an ecological macroeconomic model for the Danish economy that can link the economic and financial system with some key aspects of the climate. To do so, we combine Stock-flow-Consistent approach (SFC) with Input-Output tables (IO) to build a hybrid model, which we call Ecological Stock-Flow-Consistent Input-Output model (E-SFCIO). Most parameters of the model are estimated using time series data from 1995 to 2019, after which, we carry out simulations. We find that the model (with some minor adjustments) can replicate the dynamics of our key variables pertaining to the economy, financial system, and climate. To further validate the model, we analyse the response of the economy to various shocks, finding that it can capture the stylised facts. The model offers a foundation for providing a reasonable assessment of the climate policies to the relevant stakeholders.

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1. Introduction

In recent years, the issue of green transition and sustainable development has played a central part in shaping national policies. The challenges inherent in the path towards green transition have garnered considerable attention from various research disciplines, ranging from scientific research - testing new technologies - to social sciences, examining the complex and multifaceted interactions between society and environmental sustainability. Within this discourse, significant emphasis is placed on the interplay between economic factors and climate targets – the understanding of which remains incomplete due to the complex nature of this relationship. As such, policymakers are striving to improve their understanding of the economic costs associated with climate policies.

Denmark, like several other countries, is faced with similar challenges. The country has set ambitious targets for reducing greenhouse gas emissions, aiming for a 70% reduction by 2030 compared to 1990 levels, surpassing the European climate law's target of 55%. However, current measures are projected to achieve only a 63.1% reduction by 2030, resulting in a shortfall of 6.9%, equivalent to 5.5 million tons of CO2E (DEA 2023). There is a clear lack of consensus amongst various stakeholders on how to achieve these targets and whether cutting emissions at a faster rate is economically feasible.

To guide policy decisions on this matter, the Danish government relies, at least partly, on the insights of the Green Reform Model (Kirk et al. 2024), which is the workhorse for studying the interaction between climate and the Danish economy. This model is built on the principles of computable general equilibrium (CGE), featuring profit maximisation by producers (firms) and utility maximisation by consumers (households). The model is supply-driven, consisting of 52 production industries and 26 energy types (Kirk et al. 2024).[1] While the model provides a very detailed description of the real economic activity, the question of whether this description is a realistic representation of the economy is subject to discussion. However, a notable shortcoming of this model is its failure of providing a realistic representation of the financial aspects of the economy, something that is considered to be quite crucial in this discussion (Pollitt and Mercure 2019). The financial system, through ad hoc assumptions, is simplified to the extent that it is almost irrelevant. This invites the same criticism that post-Keynesians have directed at supply-side models in general (Lavoie and Godley 2001; Godley and Lavoie 2006).

In this paper, we aim to develop an ecological macroeconomic model that can link the economic and financial system with some key aspects of the climate. Our main goal is to provide a holistic overview of the issue using a model that, apart from capturing the stylized facts, is capable of evaluating the economic and financial risks associated with climate policies of socio-economic nature. To link the economy, environment and the financial system, we combine Stock-flow-Consistent approach (SFC) with Input-Output tables (IO) to build a hybrid model, which we call Ecological Stock-Flow-Consistent Input-Output model (E-SFCIO). The integration of these two approaches allows us to connect final demand with supply and inputs of the firms. A key element of our model is the inclusion of a financial market; sectoral wealth and debt are modelled in more detail, providing a more nuanced understanding of who is paying for the green transition. When solving the model, most parameters are estimated using time series data from 1995 to 2019, after which, we carry out simulations, finding that the model is capable of replicating the dynamics of our key variables pertaining to the economy, financial system, and climate. To further validate the model, we analyse the response of the economy to various shocks, finding that it can capture the stylised facts of the Danish economy.

We believe our proposed model will serve as a foundation for providing a reasonable assessment of the climate policies to the relevant stakeholders. Our model structure is based on a social accounting matrix where the most relevant transactions are registered (intermediate and final consumption, income and tax payments, transfers, etc.) on a whom-to-whom basis with a coherent representation of their corresponding financial transactions and environmental impacts. The detailed representation of the social and economic structure enables the analysis of the impact of climate policies on

[1] Firms, while adhering to a generic CES (Constant Elasticity of Substitution) production function, optimize between two complementary inputs namely, capital and energy.

income and wealth distribution - an aspect that, striking as it may seem, is beyond the scope of many macroeconomic models. Moreover, it allows to design and test fine-tuned sector-specific (or industry-specific) policies. Furthermore, the explicit modeling of the main financial assets and liabilities of the key sectors of the economy allow for a coherent description of the multiple ways of financing climate policies, as well as the risks inherent in their implementation. Our model, strongly influenced by post-Keynesian theory, is built on the notion of demand-led growth and excludes discount rates and damage functions. Its main focus will be the testing of the feasibility and joint coherence of multiple climate policies, but not the estimation of the feedback effects from the environment to the economy.[2]

The remainder of the paper is organized as follows. In section 2, we provide a brief overview of the Danish climate targets. In section 3, we offer a background discussion of the type of model developed in this paper. In section 4, will provide a detailed description of the databank constructed for the model. In section 5, we present the model equations and the overall structure of the model. In section 6, we carry out model evaluation, comparing the model simulations with observed data. In the same section, we show how the model response as we perform 3 simple shocks to the model. Section 7 concludes this paper.

2. The Danish Climate goals

Over the last decade, sustainable development and green transition have been at the heart of policy discussions. In 2015, 196 countries around the world joined the Paris Agreements with the aim of reducing the emission of greenhouse gases (GHG) significantly in such a way that the 2°C target (of atmospheric temperature increase above pre-industrial levels) is met by 2050 (UNFCCC 2015). In 2020, the Danish Parliament signed the Climate Law, according to which the emission of GHG must be lowered by 50% in 2025 and by 70% in 2030 (compared to the level in 1990 which was approx. 78 million tons[3]). This target is more ambitious than the target set by the European climate law, which aims for a 55% reduction in GHG emissions by 2030 compared to 1990 levels. Furthermore, the Danish climate law, in line with the European climate law, has set a long-term goal

[2] The neglect of feedback effects does not imply that they are considered unimportant. Rather, the scope of this project is to build a first version of an E-SFCIO for Denmark. Once this model is running and producing reliable results it will be possible to move to a second stage, where the feedback effects are incorporated into the model. It is worth mentioning that there is no scientific consensus about how these feedback effects should be modeled, which drives us to conclude that the investigation of these phenomena constitutes a research project on its own.

[3] Here, the estimate is based on net emissions within the Danish territory (excl. Greenland and Faroe Islands) and includes LULUCF.

of making the country climate-neutral (i.e., net GHG emissions equal to zero) by 2050 (EPRS 2021).

Apart from its own national targets, Denmark has an obligation of reducing emissions in specific sectors not included in the ETS (Emission Trading System) as part of the EU targets for 2030. More specifically, the 2023 revised version of EU's Effort Sharing Regulation (ESR) in 2023 obliges Denmark to reduce GHG emissions in non-ETS sectors – comprising agriculture, transport (excl. aviation), building heating, small industries, and waste – by 50% collectively in 2030 (compared to 2005 levels).

While assessing Denmark's path towards green transition, the Danish Energy Agency (DEA) and Danish Council on Climate Change (DCCC 2023, 2024)[4] in their assessments have repeatedly underscored the significant challenges that Denmark faces in achieving both its national as well as EU targets. According to 2022 statistics, the net domestic emission was 43.3 million tons, suggesting a 41.7% reduction compared to the 1990s. The country still needs a significant reduction of approximately 20 million tons in the remaining period to meet the 2030 national target. A recent assessment by DEA (2023) indicates that Denmark will most likely miss its targets unless significant reductions in agriculture and transport sectors are achieved, as both of these sectors collectively contribute to more than half of the total net emissions.[5] More specifically, with the current measures in place, it is projected that Denmark will achieve 63.1 percent reduction by 2030 (leaving a gap of 6.9 percent, which corresponds to 5.5 million tons of CO2). To close the remaining gap, the government has recently reached an agreement including taxation on CO2-equivelant (CO2E) emissions emitted by the agricultural sector starting from 2030. How such policies will affect the economy, are the type of questions, we would like to address using our model.

3. Combining Stock-flow models with Input-Output tables

Stock-Flow Consistent models gained a lot of attention after the publication of Godley and Lavoie (2006), and more so after the 2008 crisis. The approach offers a consistent methodology that relates stocks and flows by way of social accounting and flow-of-fund matrices. Since the traditional national accounts are now complemented by the new System of Environmental-Economic

[4] The Danish Council on Climate Change (DCCC) is assigned with the task of advising the government on achieving its intended targets.
[5] According to 2022 statistics, agriculture contribute 27% and transport contribute 29% to the total net GHG emissions in CO2 equivalents.

Accounting (SEEA) data, this makes SFC approach a natural candidate for the integration of environmental issues into the economic models. The SFC and IO frameworks have independently co-existed for a long time, but the approach of integrating the two is a recent development. While the standalone SFC framework offers a comprehensive perspective on economic and financial dynamics, the integration of IO is particularly useful for addressing pressing climate-related issues. This combined SFC-IO methodology presents a promising research avenue and is increasingly attracting attention within the field of ecological economics. The number of existing studies using SFC-IO approach is very limited. Most studies in this regard have used SFC-IO setup to build either fully or partly theoretical models (see, e.g., Berg et al 2015; Naqvi 2015; Jackson & Jackson 2021; Dunz et al. 2021).[6] Full-fledged empirical models based on SFC-IO approach for individual countries are still in the making. At the time of this writing, several projects are under development while only a very small number of studies are currently available in the literature (Valdecantos 2021).[7] Thus, our work is also a contribution to the emerging literature in ecological macroeconomics.

We extend the existing SFC framework in three ways: i) by integrating a full Input-Output (IO) table into the modeling framework, ii) by integrating environmental aspects (such as energy usage and supply by each industry) into the analysis, and iii) by introducing a relationship between energy usage (in physical units) and economic activity while capturing the resulting GHG emissions.

We now procced to providing a formal description of the steps involved in constructing this model as well as the overall structure of the model. In our presentation, we reserve the term "sector" to describe institutional sectors of the economy namely, households, non-financial corporations, financial corporation, government, and rest of the world. We use the term "industry" to describe different industries involved in production.

4. Data requirements for the model

In this section, we will describe the construction of the databank used in E-SFCIO model for Denmark. For the industry level, we use the annual input-output data, and for the sectoral level, we use the annual national accounts (including both transactions and balance sheets). To implement

[6] In the Life cycle assessment literature (LCA) there has also been theoretical contributions combining the Input-output analysis with Stock-Flow Consistent modelling (Almeida et al. 2022).
[7] Ongoing projects include empirical SFC-IO models for countries like Greece, Italy, Argentina, and others.

ecological aspects into the model, we also include energy and emission accounts for the Danish economy.

4.1. Input-Output data

We use IO data from Statistics Denmark for the period 1995-2019 and divide the production sector into nine industries (see appendix 8.1 for more information about the industries). In Table 1, we provide a general representation of IO flows used in our model. The inter-industry flows are captured via a 9 by 9 matrix. The final demand block consists of consumption, public consumption, investment, change in inventories[8], and exports. Households' consumption basket consists of a wide range of products, where a detailed classification is carried out for the food products. The final demand flows are captured via a 11 by 9 matrix. We have classified the IO table into different blocks, each representing a matrix. Understanding the dimensions of these different blocks will play a crucial role in understanding the equations in this paper. We encourage the reader to take a moment to fully understand the blocks (highlighted in different colours) and their dimensions.

Table 1: Input-Output Matrix (general representation)

<table>
<tr><td></td><td colspan="9"></td><td colspan="7">Cons</td><td colspan="4"></td><td></td></tr>
<tr><td>General</td><td>Agriculture</td><td>Foresty</td><td>Fishery</td><td>Mining and Quarry</td><td>Manu. Food</td><td>Energy prod.</td><td>Energy intensive</td><td>Financial corp.</td><td>Other industries</td><td>Other cons</td><td>Bread</td><td>Meat cons</td><td>Fish Cons</td><td>Diary cons</td><td>Fruit and vegies cons</td><td>Other food cons</td><td>Gov</td><td>Inv</td><td>Inventories</td><td>Ex</td><td>Totals</td></tr>
<tr><td>Flow: Agriculture</td><td colspan="9" rowspan="9">Intermediate flows</td><td colspan="11" rowspan="9">Final demand for domestic products</td><td rowspan="9">Total Production</td></tr>
<tr><td>Flow: Foresty</td></tr>
<tr><td>Flow: Fishery</td></tr>
<tr><td>Flow: Mining</td></tr>
<tr><td>Flow: Manufacturing of food products</td></tr>
<tr><td>Flow: Energyproduction and refineries</td></tr>
<tr><td>Flow: Other energy intensive industries</td></tr>
<tr><td>Flow: Financial corporations</td></tr>
<tr><td>Flow: Other industries</td></tr>
<tr><td>Im Flow: Agriculture</td><td colspan="9" rowspan="9">Specified imports used in domestic production</td><td colspan="11" rowspan="9">Final demand for specified imports</td><td rowspan="9">Total specified imports</td></tr>
<tr><td>Im Flow: Foresty</td></tr>
<tr><td>Im Flow: Fishery</td></tr>
<tr><td>Im Flow: Mining</td></tr>
<tr><td>Im Flow: Manufacturing of food products</td></tr>
<tr><td>Im Flow: Energyproduction and refineries</td></tr>
<tr><td>Im Flow: Energy intensive industries</td></tr>
<tr><td>Im Flow: Financial corporations</td></tr>
<tr><td>Im Flow: Other industries</td></tr>
<tr><td>Unspecified imports</td><td colspan="9">Unspecified imports used in domestic production</td><td colspan="11">Final demand for unspecified imports</td><td>Total unspecified imports</td></tr>
<tr><td>Im duties total</td><td colspan="9">Import duties associated with domestic production</td><td colspan="11">Import duties associated with final demand</td><td>Total import duties</td></tr>
<tr><td>Commodity taxes</td><td colspan="9" rowspan="5">Value added in production</td><td colspan="11" rowspan="5">Value added through final demand</td><td rowspan="5">Total value added</td></tr>
<tr><td>VAT</td></tr>
<tr><td>Other production taxes</td></tr>
<tr><td>Compensation of Emp</td></tr>
<tr><td>Gross operating surplus and mixed income</td></tr>
<tr><td>Totals</td><td colspan="9">Total outlays</td><td colspan="11">Total aggregate demand (C+I+G+X)</td><td></td></tr>
</table>

We now move from a general representation of the IO matrix to the specific case of Denmark. Table 2 is a representation of an IO-table using Danish data (in nominal values) for 2019. When moving across the table horizontally, the flows represent the production of each industry. For example, the first row shows the production of agricultural sector. Note that each industry engages in two types of production, i) production of products sold as intermediate goods to other industries (which are used as inputs in production), and ii) production of final products sold to various institutional sectors (incl. rest of the world), determining final demand. If we move vertically down the table (or read the table from top to bottom), the entries (with the exception of gross operating surplus and mixed income)

[8] To simplify the model, we have added the acquisitions less disposals of valuables to the change in inventories. Both variables are held exogenous within the model and will enter the same equations whereas it will not impact any results.

represent the costs of domestic industry associated with production. For example, the first column of the table shows the cost of production in the agricultural industry. The cost of the industry consists of three categories, i) domestic and imported inputs, ii) production and value added taxes, and iii) compensation of employees. The difference between the total value of production and costs, gives us the gross operating surplus. This will be explained in more detail, when presenting the model equations in section 5.

Table 2: Input-Output Matrix of 2019

										Cons											
Year 2019	Agriculture	Foresty	Fishery	Mining and Quarry	Manu. Food	Energy prod.	Energy intensive	Financial corp.	Other industries	Other cons	Bread	Meat cons	Fish Cons	Diary cons	Fruit and vegies cons	Other food cons	Gov	Inv	Inventories	Ex	Totals
Flow: Agriculture	8.892	0.033	0.000	0.000	27.700	0.803	0.008	0.001	17.082	1.842946	0	0.071146	0.000103	0.432373	1.94455	0	0.000014	0.194661	-0.519376	18.7464	77.232
Flow: Foresty	0.661	1.554	0.000	0.014	0.007	0.514	0.337	0.000	0.253	0.280519	0	0	0	0	0	0	0.458835	0.003029	0.47601	0.37555	4.933
Flow: Fishery	0.017	0.000	0.019	0.000	0.841	0.000	0.011	0.000	0.053	0.04215	0	0	0.051686	0	0	0	0	0.000514	0.005022	3.99022	5.031
Flow: Mining	0.193	0.000	0.000	3.178	0.031	11.303	0.787	0.000	1.319	0.210944	0	0	0	0	0	0.004645	0	0.148399	-0.334196	11.6924	28.532
Flow: Manufacturing of food products	9.673	0.001	0.902	0.002	10.121	0.468	1.583	0.011	7.979	3.894332	6.0422	6.96811	0.434078	0.170854	0.728751	3.041834	0	0.419254	0.259052	61.544	114.241
Flow: Energyproduction and refineries	1.518	0.057	0.057	0.104	1.327	4.475	2.547	0.589	19.503	27.25389	0	0	0	0	0	0	0	0.890297	0.856937	19.0262	78.206
Flow: Other energy intensive industries	0.981	0.009	0.029	0.285	2.383	0.844	29.894	0.359	62.424	10.88993	0.0156	0.029519	0	0.002148	0.001292	0.075708	0.054533	17.61044	4.617358	225.126	355.630
Flow: Financial corporations	5.519	0.148	0.155	0.224	0.643	1.063	1.453	39.695	58.479	65.98409	0	0	0	0	0	0	0	5.782596	0.001123	29.1563	208.302
Flow: Other industries	12.234	0.755	0.948	3.310	17.248	7.791	75.008	29.146	783.878	626.7787	3.5188	6.961754	2.338726	10.57305	6.842517	9.837666	549.98098	331.0943	3.979379	711.487	3193.711
Im Flow: Agriculture	0.780	0.025	0.000	0.000	2.313	0.000	0.015	0.002	1.808	1.561244	0	0.019771	0.000078	0.08859	3.140565	0.038273	0.000042	0.000853	-0.554041	3.79952	13.039
Im Flow: Foresty	0.001	0.004	0.000	0.010	0.004	0.359	0.234	0.000	0.127	0.064129	0	0	0	0	0	0	0	0	-0.037654	0.03191	0.796
Im Flow: Fishery	0.034	0.000	0.000	0.000	3.746	0.000	0.045	0.000	0.382	0.004496	0	0	0.370024	0	0	0	0	0	0.018918	2.21058	6.811
Im Flow: Mining	0.053	0.000	0.000	0.668	0.084	18.802	0.517	0.000	0.495	0.033376	0	0	0	0	0	0.032941	0	0.006846	-0.291622	0.32249	20.724
Im Flow: Manufacturing of food products	3.589	0.001	0.228	0.006	13.406	0.408	1.965	0.012	9.923	8.567134	2.7082	5.552527	0.715708	0.141345	2.787639	4.098315	0.077937	0.000019	0.166402	15.2679	69.620
Im Flow: Energyproduction and refineries	1.261	0.040	0.459	0.116	0.234	0.527	0.869	4.283	5.478	3.608038	0	0	0	0	0	0	0	0	0.190549	12.7294	29.795
Im Flow: Energy intensive industries	2.265	0.002	0.013	0.456	2.647	2.993	41.390	0.171	67.040	10.94577	0.0177	0.031173	0.006828	0.028241	0.380785	0.133631	3.501664	2.055461	1.891295	54.282	190.251
Im Flow: Financial corporations	0.052	0.002	0.006	0.055	0.114	0.121	0.660	3.259	16.106	3.86214	0	0	0	0	0	0	0	0	0.013028	0.91736	25.168
Im Flow: Other industries	3.145	0.071	0.127	0.835	6.436	1.168	29.342	4.342	238.451	76.10548	0.0169	0.014705	0	1.487537	0	0.593005	1.367655	80.97616	2.512439	124.965	571.957
Unspecified imports	0.007	0.011	0.000	0.216	0.068	0.082	0.620	5.059	212.401	-14.5317	0	0	0	0	0	0	0.000	0.287	0.000	59.556	263.776
Im duties total	0.035	0.000	0.002	0.003	0.146	0.003	0.208	0.003	0.700	0.536177	0.0229	0.012156	0.008336	0.000284	0.031	0.022	0.002	0.110	0.026	1.259	3.128
Commodity taxes	1.109	0.067	0.173	0.083	0.271	-0.004	0.556	0.064	13.551	45.511	0.075	0.000	0.000	0.000	0.041	2.488	0.069	9.584	0.605	-1.175	73.068
VAT	0.013	0.003	0.003	0.004	0.028	0.018	0.420	5.734	46.572	105.731	3.017	4.776	0.953	3.137	3.835	4.890	2.078	41.702	0.267	0.000	223.180
Other production taxes	-6.198	0.070	0.011	0.098	0.426	-1.310	1.003	8.429	21.979												24.509
Compensation of Emp	9.947	1.331	0.863	4.261	17.573	7.470	57.847	61.403	1034.140												1194.836
Gross operating surplus and mixed income	21.454	0.749	1.036	14.604	6.444	20.306	108.312	45.741	573.588												792.234
Totals	77.232	4.933	5.031	28.532	114.241	78.206	355.630	208.302	3193.711	979.176	15.435	24.437	4.879	16.062	19.732	25.256	557.590	490.867	14.149	1355.310	7568.710

Note: All the flows from 2019 are presented in nominal values

The products of each industry sold to the households are categorised into 7 sub-categories. That is, from the perspective of households, the consumption basket consists of 7 types of products, supplied by 9 industries. It is worth noticing that a given product in the consumer basket can be supplied by more than one domestic and foreign industry. For example, bread products are provided by 3 different types of domestic and foreign industries. The categorisation of each industrial product into sub-categories is only carried out for the consumption basket of the household sector, as private consumption constitutes the major share of aggregate demand. The detailed sub-categorisation is strategically performed for food items, as taxing emissions in the agricultural sector has lately received a lot of attention. For other sectors, we retain a certain level of aggregation and account for spending on goods according to the type of industries.

The input output table shown in table 2 provides the basis of the nominal industry level data used within the model. Before moving to the presentation of price indices used to deflate the nominal values in the IO-table, the distinction between nominal and real values should be made clear. In the rest of this paper, we denote nominal variables with upper case letters and deflated variables with lower case letters. However, it should be noted that matrices (both nominal and real) will be written in uppercase and bold, but we will use 2010 as a superscript to denote real matrices (2010 representing the index and weight reference year used in calculating price deflators). Vectors (both real and nominal) will be written in lower case and bold, and again, we will use 2010 as a superscript to denote real vectors.

4.1.1. Price indices

To distinguish between unit changes and price changes in the model, we calculate the IO table for each year in deflated values using 2010 as the base year.[9] We deflate the following flows: domestic intermediate flows (domestic inputs), import flows used as inputs, and all aggregate demand flows (which also include imports and exports). We then use these deflated values to calculate technical coefficients in section 4.1.2.

To calculate price indices, we use the data provided by Statistics Denmark where IO-tables are both available in current and last year's prices, which allows us to calculate a price index using the Paasche price index technique. For each variable (V) and each time period (t), we calculate the change in price (P), by dividing the value in current prices ($V_t P_t$) by the value of last year's prices ($V_t P_{t-1}$). [10]

$$R_t = \frac{V_t P_t}{V_t P_{t-1}} = \frac{P_t}{P_{t-1}} \qquad \text{(equation 1)}$$

After calculating R_t (which represents the price growth factor), we define a price index by setting a value of 1 in the base year, 2010. To calculate the price index after 2010, we multiply the base value with the price growth factor (R_t) while moving forward each period; to calculate the price index before 2010, we multiply the base value with the *inverse* of the price growth factor while moving backwards each period. This way, we create an index that allows us to calculate the deflated values of a variable in 2010 prices. We simplify the set-up by using an aggregate price index for each domestic and foreign industry. In total, we compute 19 price indices as shown by each row of Table 3.[11] Here, py_t^1 is the producer price index for the total production of industry 1 and pm_t^1 is the

[9] The reason we use 2010 as the base year is because Statistics Denmark provides most aggregate variables in both weighted prices (using 2010 as a base year) and current prices.

[10] Statistics Denmark does not provide other production taxes, compensation of workers, and gross operating surplus in last year prices. Therefore, we do not calculate the value-added block in 2010 prices. Still, we are able to use the IO-setup to first calculate output in 2010 prices and then use price deflators to also calculate all components of the IO-table in nominal prices. Thus, we are able to match total output with total outlays for the nominal values, ensuring a consistent IO-table.

[11] Additionally, we calculate price indices for value-added taxes and commodity taxes for the final demand part. We do this to calculate the aggregate final demand variables in deflated values in the model.

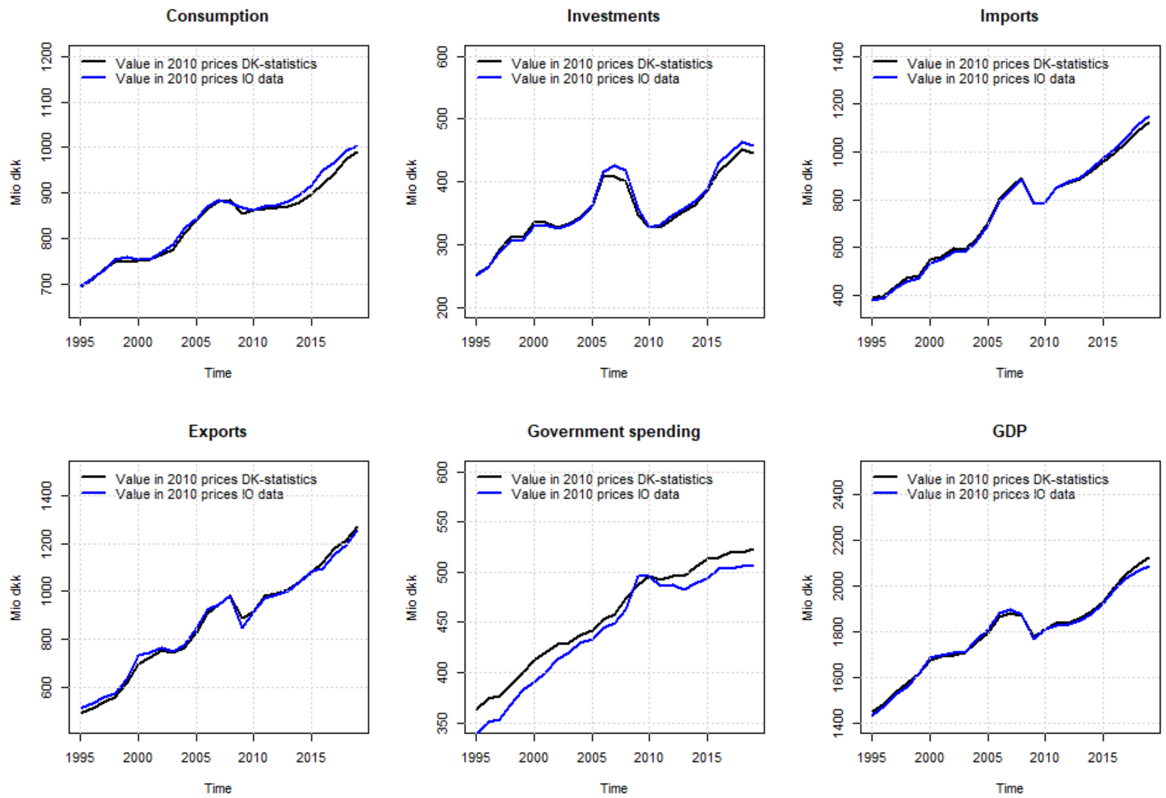
import price index for industry 1 also in producer prices. The price index pm_t^{un} is used to deflate the imports which can not be associated with a specific industry whereas it is classified as unspecified imports.

Table 3: Price indices in IO.

										Cons											
Price deflators	Agriculture	Foresty	Fishery	Mining and Quarry	Manu. Food	Energy prod.	Energy intensive	Financial corp.	Other industries	Other cons	Bread	Meat cons	Fish Cons	Diary cons	Fruit and vegies cons	Other food cons	Gov	Inv	Inventories	Ex	Totals
Flow: Agriculture	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1	py_1
Flow: Foresty	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2	py_2
Flow: Fishery	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3	py_3
Flow: Mining	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4	py_4
Flow: Manufacturing of food products	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5	py_5
Flow: Energyproduction and refineries	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6	py_6
Flow: Other energy intensive industries	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7	py_7
Flow: Financial corporations	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8	py_8
Flow: Other industries	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9	py_9
Im Flow: Agriculture	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1	pim_1
Im Flow: Foresty	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2	pim_2
Im Flow: Fishery	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3	pim_3
Im Flow: Mining	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4	pim_4
Im Flow: Manufacturing of food products	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5	pim_5
Im Flow: Energyproduction and refineries	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6	pim_6
Im Flow: Energy intensive industries	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7	pim_7
Im Flow: Financial corporations	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8	pim_8
Im Flow: Other industries	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9	pim_9
Unspecified imports	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un	pim_un
Im duties total																					Nominal value
Commodity taxes																					Nominal value
VAT										Not deflated											Nominal value
Other production taxes																					Nominal value
Compensation of Emp																					Nominal value
Gross operating surplus and mixed income																					Nominal value
Totals										pc_oth	pc_110	pc_120	pc_130	pc_140	pc_160	pc_180	p_g	p_i	p_i	p_x	

Overall, we find that our constructed price indices match the ones published by Statistics Denmark quite well. For the purpose of illustration, in Figure 1, we show how our calculated price indices for key variables such as final consumption, exports, investments, and government spending compares to the values provided by Statistics Denmark. Apart from the level difference in the deflators for government consumption, our indices match the trend and fluctuation of the data.

Figure 1: Real final demand components compared to observed data.



To create an index for household consumption, we first compute aggregate real consumption using equation 2:

$$c_t^{agg} = \sum_{n=1}^{9}\left(\frac{\sum_{p=1}^{7} C_{dom,t}^{n\,p}}{py_t^n}\right) + \sum_{n=1}^{9}\left(\frac{\sum_{p=1}^{7} C_{im,t}^{n\,p}}{pm_t^n}\right) + \frac{M_{cons,t}^{duty,n}}{pm_{duty,t}} + \frac{C_{uim,t}}{pm_{uim,t}} + \frac{\sum_{p=1}^{7} C_{ctax,t}^{p}}{pc_{ctax,t}} + \frac{\sum_{p=1}^{7} C_{VAT,t}^{p}}{pc_{vat,t}} \quad \text{(equation 2)}$$

Real aggregate consumption is calculated by deflating each consumption component by its relevant price deflator. The first term of this equation indicates the summation of household consumption values across each product type, divided by the price of the corresponding industry's price (py_t^n), and then summing up these real values across all industries to get the aggregate consumption. The second term of the equation follows the same logic but for imported goods.

$C_{dom,t}^{n\,p}$ is the nominal consumption of different goods type p supplied by the 9 domestic industries, each divided by the relevant price deflator py_t^n. The notation given by $\sum_{p=1}^{7} C_{im,t}^{n\,p}$ represents nominal consumption of different goods type p supplied by the 9 foreign industries, each divided by the relevant price deflator pm_t^n. We also include import duties ($M_{cons,t}^{duty}$) paid by the consumer and divide it by the respective price deflator ($pm_{duty,t}$). $C_{uim,t}$ are the unspecified nominal imports, deflated using the price deflator for unspecified imports ($pc_{uim,t}$). Finally, commodity taxes $C_{ctax,t}^{p}$ and value added taxes $C_{vat,t}^{p}$ are deflated using their respective price deflators $pc_{ctax,t}$ and $pc_{vat,t}$. It is important to highlight that the price deflators for import duties, commodity taxes and value added taxes are merely computed to carry out the calculations in equation 2, beyond which, they have no role as these variables will only be used in nominal terms for our analysis. Therefore, we do not represent them in the price dynamics represented in Table 3. Note that equation 2 without price denominators simply equals total nominal consumption (C_t^{agg}).

We can now calculate the price deflator for aggregate consumption (pc_t^{agg}) as follows:

$$pc_t^{agg} = \frac{C_t^{agg}}{c_t^{agg}} \qquad \text{(equation 3)}$$

We follow the same strategy and calculate a price index for each aggregate demand component namely, investment deflator (pin_t^{agg}), inventory deflator ($pinvent_t^{agg}$), government spending deflator (pg_t^{agg}), exports deflator (px_t^{agg}), imports deflator (pm_t^{agg}), and finally, the GDP deflator (py_t^{agg}).

4.1.2. Technical coefficients

In this section, we perform the calculations of technical coefficients. Before calculating the technical coefficients, we deflate the IO data using the price indices presented in the previous section. Our strategy is to separate the effect of price changes from the effect of unit changes in IO setup. Using the price deflators presented in Table 3, we create a matrix $\boldsymbol{Z_{dom,t}^{2010}}$ which consists of domestically produced inputs for production but in 2010 prices. Similarly, we create a final demand matrix $\boldsymbol{F_{dom,t}^{2010}}$ in 2010 prices. We can now set up the following system of equations:

$$\boldsymbol{Z_{dom,t}^{2010} i_9 + F_{dom,t}^{2010} i_{11} = prod_t^{2010}} \qquad \text{(equation 4)}$$

The intuition behind equation 4 is straightforward: the sum of intermediate goods and final demand gives us the total value of domestic production. To understand the matrix representation of equation 4, it is important to understand the dimensions of the different blocks in the IO matrix. Even though Table 2 is a matrix of nominal values, we can still make use of it for the purpose of explaining the dimension of our matrices. Our IO table is categorised into multiple matrices; $\boldsymbol{Z_{dom,t}^{2010}}$ matrix (which has 9 X 9 dimension, represented by the top-left, highlighted in light blue colour) is converted into a column vector by multiplying it with a column vector of ones $\boldsymbol{i_9}$. The dimension of the column vector is 9 X 1 as we have 9 industries (or columns). Similarly, the final demand matrix $\boldsymbol{F_{dom,t}^{2010}}$ (which is 9 X 11 represented by the top-right, highlighted in blue) is converted into a column vector by multiplying it with a column vector of ones $\boldsymbol{i_{11}}$ (here we use 11 X 1 column vector). The sum of the two resultant column vectors (intermediate goods and final demand) gives us a vector of total production, consisting of 9 rows (which is the last column of Table 2 highlighted in dark blue). Here, each row represents a corresponding industry, e.g., the first value of the total production vector will represent the total (real) production by the agricultural sector (the nominal value is 77.232).

Since the demand for inputs (or inputs purchased) by an industry, amongst others, is determined by the total production (or demand for the products) of that industry, we assume a linear relationship between inputs and production. More specifically, this relationship is captured through technical coefficients. The matrix of technical coefficients ($\boldsymbol{A_t^{2010}}$) can be written as follows:

$$\boldsymbol{A_t^{2010}} = \begin{bmatrix} a_t^{11} & \cdots & a_t^{1n} \\ \vdots & \ddots & \vdots \\ a_t^{n1} & \cdots & a_t^{nn} \end{bmatrix} \qquad \text{(equation 5a)}$$

where each element of this matrix is calculated as follows: [12]

$$a_t^{i\,n} = \frac{z_t^{in}}{prod_{dom,t}^{n}} \qquad \text{(equation 5b)}$$

In the above, $z_t^{i\,n}$ denotes total inputs, including both the real value of domestic inputs $z_{dom,t}^{i\,n}$ and the real value of imported inputs $z_{im,t}^{i\,n}$ purchased by an industry *n* from industry *i* (thereby adding the light blue matrix and the light orange matrix together in Table 2).

$$z_t^{i\,n} = z_{dom,t}^{i\,n} + z_{im,t}^{i\,n} \qquad \text{(equation 5c)}$$

Re-arranging equation 5b, we can write equation 5d which relates total production in industry *n* to the input requirements from industry *i*:

$$z_t^{i\,n} = a_t^{i\,n} * prod_{dom,t}^{n} \qquad \text{(equation 5d)}$$

The total inputs in matrix form can be expressed as:

$$\boldsymbol{Z_t^{2010}} = \begin{bmatrix} z_t^{11} & \cdots & z_t^{1n} \\ \vdots & \ddots & \vdots \\ z_t^{n1} & \cdots & z_t^{nn} \end{bmatrix} \qquad \text{(equation 5e)}$$

[12] Here, we attempt to be consistent with the standard matrix notations where elements of a matrix **X** are usually represented by $X_{i\,j}$, where *i* denotes rows and *j* denotes columns. Similarly, in our notations $Z^{n\,i}$ implies that *n* represents rows and *i* represents columns. It is important to emphasize that when we use the notation $Z^{i\,n}$, we do not mean to switch the rows and columns, but in that case, *i* represents rows and *n* represents columns.

Which can then be split into a matrix of domestically produced intermediate good $\boldsymbol{Z_{dom,t}^{2010}}$ and foreign produced intermediate goods $\boldsymbol{Z_{im,t}^{2010}}$. Section 5.2.6 describes how this distinction between domestically and foreign produced intermediate goods are carried out.

$$\boldsymbol{Z_{dom,t}^{2010}} = \begin{bmatrix} z_{dom,t}^{11} & \cdots & z_{dom,t}^{1n} \\ \vdots & \ddots & \vdots \\ z_{dom,t}^{n1} & \cdots & z_{dom,t}^{nn} \end{bmatrix} \qquad \text{(equation 5f)}$$

$$\boldsymbol{Z_{im,t}^{2010}} = \begin{bmatrix} z_{im,t}^{11} & \cdots & z_{im,t}^{1n} \\ \vdots & \ddots & \vdots \\ z_{im,t}^{n1} & \cdots & z_{im,t}^{nn} \end{bmatrix} \qquad \text{(equation 5g)}$$

In a static setup, the relationship between final demand $\boldsymbol{F_{dom,t}^{2010}}$ and production $\boldsymbol{prod_t^{2010}}$ would require the calculation of the Leontief inverse,[13] but since we use a dynamic setup, we can simply use equation 4 in combination with equation 5d to calculate total production for each industry, which for industry *n* will result in equation 6 presented in section 5.1.1.

4.2. Transaction-Flow-Matrix from a sectoral perspective

Before we move further into the data requirements of the model, we find it important to first provide an overview of the transaction flows from the perspective of institutional sectors in the economy (aka Transaction-Flow-Matrix - TFM). Our model consists of 5 institutional sectors namely, households, non-financial corporations (NFC), financial corporations (FC), government, and the rest of the world (ROW).

Table 4: Transaction-Flow-Matrix

[13] In a static input-output model one should obtain the Leontief inverse to relate domestic final demand ($\boldsymbol{F_t^{2010}}$) to domestic production ($\boldsymbol{prod_t^{2010}}$). This can be done using the following equation: $\left(\boldsymbol{I_9} - \boldsymbol{A_t^{2010}}\right)^{-1} * \boldsymbol{F_t^{2010}} \boldsymbol{i_{11}} = \boldsymbol{prod_t^{2010}}$**.** Where $\left(\boldsymbol{I_7} - \boldsymbol{A_t^{2010}}\right)^{-1}$ is referred to as the Leontief inverse ($\boldsymbol{L_t^{2010^{-1}}}$).

	Households		Non-financial Firms		Financial Firms		Government		RoW		Total
	Current	Capital	Current	Capital	Current	Capital	Current	Capital	Current	Capital	
Consumption	$-C$		C								0
Investment		$-I_H$	I	$-I_{NFC}$		$-I_{FC}$		$-I_G$			0
Gov Expenditure			G				$-G$				0
Exports			X						$-X$		0
Imports			$-M$						M		0
GDP			Y								Y
Wages	WB_H		$-WB_{NFC}$						WB_{ROW}		0
Prod. Taxes			$-PTax$				$PTax$				0
Total Gross surplus			$-B2_{tot}$								$-B2_{tot}$
Gross surplus	$B2_H$		$B2_{NFC}$		$B2_{FC}$		$B2_G$				$B2_{tot}$
Net income interres	$NINT_h$		$NINT_{NFC}$		$NINT_{FC}$		$NINT_G$		$NINT_{ROW}$		0
Net income insurance	$NOIR_h$		$NOIR_{NFC}$		$NOIR_{FC}$		$NOIR_G$		$NOIR_{ROW}$		0
Net dividents	$NDIV_h$		$NDIV_{NFC}$		$NDIV_{FC}$		$NDIV_G$		$NDIV_{ROW}$		0
Net FDI	$NFDI_h$		$NFDI_{NFC}$		$NFDI_{FC}$		$NFDI_G$		$NFDI_{ROW}$		0
Social contributions	$-SCON_h$				$SCON_{FC}$		$SCON_G$		$SCON_{ROW}$		0
Social Benefits	$SBEN_H$				$-SBEN_{FC}$		$-SBEN_G$		$SBEN_{ROW}$		0
Other current transfers	OCT_H		OCT_{NFC}		OCT_{FC}		OCT_G		OCT_{ROW}		0
Income tax	$-T_H$		$-T_{NFC}$		$-T_{FC}$		T_G		$-T_{ROW}$		0
Savings	$-S_H$	S_H	$-S_{NFC}$	S_{NFC}	$-S_{FC}$	S_{FC}	$-S_G$	S_G	$-S_{ROW}$	S_{ROW}	0
Capital tranfers		CT_H		CT_{NFC}		$-CT_{FC}$		$-CT_G$		$-CT_{ROW}$	0
Others		$-NP_H$		$-NP_{NFC}$				NP_G		NP_{ROW}	0
Net lending		NL_H		NL_{NFC}		NL_{FC}		NL_G		NL_{ROW}	0
Adjustment variable		adj_H		adj_{NFC}		adj_{FC}		adj_G		adj_{ROW}	0
ΔGold						ΔG_{FC}				$-\Delta G_{ROW}$	0
ΔDeposits		$\Delta NDEP_H$		$\Delta NDEP_{NFC}$		$-\Delta NDEP_{FC}$		$-\Delta NDEP_G$		$-\Delta NDEP_{ROW}$	0
ΔSecurities		$-\Delta NSEC_H$		$\Delta NSEC_{NFC}$		$-\Delta NSEC_{FC}$		$\Delta NSEC_G$		$-\Delta NSEC_{ROW}$	0
ΔLoans		$\Delta NLOA_H$		$\Delta NLOA_{NFC}$		$-\Delta NLOA_{FC}$		$-\Delta NLOA_G$		$\Delta NLOA_{ROW}$	0
ΔEquities		$-\Delta NEQ_H$		ΔNEQ_{NFC}		$-\Delta NEQ_{FC}$		$-\Delta NEQ_G$		ΔNEQ_{ROW}	0
ΔInsurance		$-\Delta NINSU_H$		$-\Delta NINSU_{NFC}$		$\Delta NINSU_{FC}$		$-\Delta NINSU_G$		$-\Delta NINSU_{ROW}$	0
ΔDerivatives		$-\Delta NDERV_H$		$-\Delta NDERV_{NFC}$		$\Delta NDERV_{FC}$		$-\Delta NDERV_G$		$-\Delta NDERV_{ROW}$	0
ΔTrade credits		$-\Delta NTCRED_H$		$-\Delta NTCRED_{NFC}$		$\Delta NTCRED_{FC}$		$-\Delta NTCRED_G$		$-\Delta NTCRED_{ROW}$	0
Rev. Gold						G^{RV}_{FC}				$-G^{RV}_{ROW}$	0
Rev. Deposits		$NDEP^{RV}_H$		$NDEP^{RV}_{NFC}$		$-NDEP^{RV}_{FC}$		$-NDEP^{RV}_G$		$-NDEP^{RV}_{ROW}$	0
Rev. Securities		$-NSEC^{RV}_H$		$NSEC^{RV}_{NFC}$		$-NSEC^{RV}_{FC}$		$NSEC^{RV}_G$		$-NSEC^{RV}_{ROW}$	0
Rev. Loans		$NLOA^{RV}_H$		$NLOA^{RV}_{NFC}$		$-NLOA^{RV}_{FC}$		$-NLOA^{RV}_G$		$NLOA^{RV}_{ROW}$	0
Rev. Equities		$-NEQ^{RV}_H$		NEQ^{RV}_{NFC}		$-NEQ^{RV}_{FC}$		$-NEQ^{RV}_G$		NEQ^{RV}_{ROW}	0
Rev. Insurance		$-NINSU^{RV}_H$		$-NINSU^{RV}_{NFC}$		$NINSU^{RV}_{FC}$		$-NINSU^{RV}_G$		$-NINSU^{RV}_{ROW}$	0
Rev. Derivatives		$-NDERV^{RV}_H$		$-NDERV^{RV}_{NFC}$		$NDERV^{RV}_{FC}$		$-NDERV^{RV}_G$		$-NDERV^{RV}_{ROW}$	0
Rev. Trade credits		$-NTCRED^{RV}_H$		$-NTCRED^{RV}_{NFC}$		$NTCRED^{RV}_{FC}$		$-NTCRED^{RV}_G$		$-NTCRED^{RV}_{ROW}$	0
ΔNet Wealth		$-\Delta FNW_H$		$-\Delta FNW_{NFC}$		$-\Delta FNW_{FC}$		$-\Delta FNW_G$		ΔFNW_{ROW}	0

It is important to highlight that the national accounts, which provide the basis for a full empirical model, can be presented at both industry and sectoral level. When describing production in the model structure, we use the industry-level accounts (discussed in section 4.1), but when explaining the TFM in Table 4, we choose to use the presentation from the sectoral perspective. The reason is that data for variables below gross operating surplus and mixed income, such as financial income or changes in financial transactions, is unavailable at industrial level. Consequently, the current model can explain the rows below gross operating surplus only at a sectoral level. For example, the model will only explain net lending (surplus/deficit) at a sectoral level but not at an industrial level.

We now provide an explanation of how industries and sectors are connected. Note that entries above the gross operating surplus are obtained from the IO table whereas flows below the gross operating surplus are obtained from the national accounts at a sectoral level. To establish a connection between industries (IO data) and institutional sectors (sectoral national accounts), we use gross operating surplus as a binding flow. This requires identifying, what share of industrial profits (from production) falls under which sectors; in this regard, we use the 2016 matrix of *industry by sector* provided by Statistics Denmark (DST 2021), which contains the share of gross value added (GVA) from each industry allocated to the corresponding institutional sectors. For example, the matrix suggests that two-thirds (approx. 66.7 percent) of the GVA from the agricultural industry belongs to the households, one-third (approx. 33.3 percent) belongs to the NFC, whereas none belongs to the other sectors. Therefore, it is assumed that two-thirds of the gross operating surplus from the agricultural industry is owned by the household sector, whereas the remaining belongs to the NFC. This implicitly suggests that two-thirds of both the total production (which includes the sale of final consumption goods) and the total costs (part of which include paying wages) related to the agricultural industry belong to the household sector. However, to keep the presentation of TFM simple, entries in relation to consumption (as an income) and wages (as an expense) are not explicitly included for the household block.

To get the weights for each of the 9 industries, we calculate a weighted average of the shares obtained from the matrix of *industry by sector*, using the level of gross operating surplus for each industry (more information about the estimation of these shares are provided in appendix 8.2).[14] It is important to highlight that the allocation of gross operating surplus (which we at the industry level call profits) to other institutional sectors should not be confused with dividend payouts against equity holdings; that mechanism is separately captured in our framework as will be discussed.

After transforming gross operating surplus and mixed income (profits) from an industry to sectoral level, we now focus on sector-specific income and expenses. Here, we rely on the sectoral national accounts data presented by Statistics Denmark. This includes sectoral saving, which is used for capital formation (investments) and other capital transfers;[15] if a sector's savings are not enough to meet its investment expenditures and other capital transfers, the sector will run into a (sectoral) deficit, which is reported by the row called net lending in the TFM. This deficit is covered by adjustments in the balance sheet as shown by the rows below net lending, representing the changes in eight net financial stocks namely gold, deposits, securities, loans, equities, insurances, derivatives, and trade credits.

The structure of the balance sheet, although not explicitly presented, is obvious from the lower part of TFM. The net value of each financial stock is calculated as the difference between the asset and liability of the same financial stock. When calculating the change in net financial wealth (as shown by the last row of the TFM), we also account for re-evaluations of the stock of net financial stocks, represented by the 8 rows just above the net financial wealth.

4.2.1. Net financial income

One crucial aspect for the sectoral data is the calculation of rates of return for the 4 types of income on financial assets (reported in the rows under the gross operating surplus) being: net interest payments denoted by *NINT*, net income on other investments (related to pension and insurances)

[14] An alternative approach is to calculate exogenous shares of total profits received by the household, the government sector, financial corporations, and non-financial corporations based on the sectoral data and then divide out total gross operating surplus using these shares. A problem that might arise following this approach is if agricultural increase profits this will lead to an increase in total profits and based on the exogenous shares profits obtained by the sectors will increase. In reality, an increase in agriculture industry profits should only increase the profits obtained by the household sector and non-financial corporations following the shares obtained from the Matrix of industry by sectors.

[15] Here, we use exogenous shares to divide the total nominal investments across the sectors.

denoted by *NOIR*, net income on net equities denoted by *NDIV*, and finally income on net FDI denoted by *NFDI*. A crucial step in calculating these rates of returns is to ensure that incomes on financial assets in the TFM (4 income streams in our case) are linked to the holdings of financial stocks (8 financial stocks in our case). To clearly establish a link in this regard and to simplify the structure of the model, we rely on certain assumptions.[16] First, we assume that the rate of return on a given financial stock is the same across institutional sectors, e.g., the interest rate on loans is the same for all sectors. This raises the question of which sector's stocks and flows should be used to calculate the rates, which motivates our second assumption; it can be argued that, since financial corporations serve as financial intermediators, this sector should be used to calculate the rates of returns on financial stocks. In appendix 8.3, we present a detailed description of how these different rates of returns are calculated, which determine the 4 types of net financial income presented in the TFM.

We now proceed to presenting the data used in the environmental block of the model.

4.3. Environmental data

We use a mix of data sources for the environmental variables. For the energy supply and use, we collect all data from Statistics Denmark. For emissions, we have chosen to use a combination of the data from GreenREFORM model and Statistics Denmark. The appealing feature of the emission data used in GreenREFORM is that we can relate a specific type of emissions to a specific type of energy usage. In the following sections, we present this in further detail, starting with the energy accounts from Statistics Denmark.

4.3.1. Energy Accounts

For the energy accounts used in our analysis, we consider 21 different types of energy supplied and used in the economy. This data is crucial for understanding the direct energy usage patterns across different economic units, providing clear insights into which energy sources are most utilized by each industry. Table 5 provides an overview of the Danish energy accounts in 2019, expressed in physical units.

Table 5: Energy supply and usage in 2019 by each industry (in million Gigajoules)

[16] Our assumptions are mostly dictated by the lack of transparency in the data.

Energy accounts 2019	Coil	Oilp	RefG	GasT	Fgas	FGasBunk	DieT	DieTBunk	SGasBunk	NGasExt	NGasCons	CC	Waste	RE	Straw	FW	WP	BioG	BBB	El	Dheat	Total
Supply																						
Agriculture	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18	0	0	0	0	0	0	18
Foresty	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	35.9	0	0	0	0	0	35.9
Fishery	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mining	215.7	0	0	0	0	0	0	0	0	118.5	0	0	0	0	0	0	0	0	0	0	0	334.2
Manufacturing of food products	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0	0	0.1
Energy production and refineries	0	126.1	16.6	91.5	9.8		97.2	0	0	0	139.9	0	0	0	0	0	0	16.6	7.7	97.6	103.7	706.7
Energy intensive industries	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	5.7	26.8	34.5
Final corporations	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Other industries	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Imported energy	207.6	142.4	0	16.8	43.4	26.9	0	23.8	497.2	43.9	0	62.4	5.7	0	0	9.1	57.2	0	2	52.4	0.1	1190.9
Waste used as input	0	0	0	0	0	0	0	0	0	0	0	0	42.9	0	0	0	0	0	0	0	0	42.9
Renewable energy used as input	0	0	0	0	0	0	0	0	0	0	0	0	0	76.1	0	0	0	0	0	0	0	76.1
Total supply	423.3	268.5	16.6	108.3	53.2	26.9	97.2	23.8	497.2	162.4	139.9	62.4	48.6	76.1	18	45	59.2	16.6	9.8	155.7	130.6	2439.3
Usage																						
Agriculture	0	11.7	0	0.4	0	0	2.5	0	0	0	1.3	0.1	0.2	0.9	2	0	0	0.3	0.2	6.5	1.5	27.6
Foresty	0	0.4	0	0.1	0	0	0.4	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0	0.93
Fishery	0	4.7	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0.3	0	5.1
Mining	0	0.9	0	0	0	0	0.1	0	0	19.8	0.7	0.2	0	0	0	0.5	0.1	0.1	0	0.3	0	22.7
Manufacturing of food products	0	1.8	0	0.1	0	0	0.7	0	0	0	7	1.2	0	0.5	0	0.2	0	1.2	0.04	6.3	0.5	19.54
Energy production and refineries	328.7	2	16.6	0.2	0.6	0	0.4	0	0	139.8	28.5	33.5	8.8	64	13.1	28.8	34.9	9.2	0.2	5.8	0.6	715.7
Energy intensive industries	0	9.3	0	0.1	0	0	2.5	0	0	0	11	3	39.2	0.8	0	0.9	0.3	1.3	0.2	12.8	1.7	83.1
Final corporations	0	0	0	0.1	0	0	0.4	0	0	0	0.3	0	0	0	0	0	0	0	0.03	0.7	0.7	2.23
Other industries	0	22	0	5.2	19.8	26.9	63.1	23.8	497.2	0	13.5	0	0.4	1.1	0	0.2	2.4	1.9	5.1	48.7	31.8	763.1
Exports	107.7	218	0	51	12.3	0	0	0	0	0	41	2.7	0	0	0	1	18.5	0	0	31.6	0	483.8
Change in inventories	-13.1	-10	0	-0.4	20.5	0	0	0	0	0	13.2	21.7	0	0	0	-3	-13	0	0	0	0	15.9
Distribution losses	0	0	0	0	0	0	0	0	0	2.8	0.2	0	0	0	0	0	0	0	0	5.7	26.1	34.8
Households	0	7.7	0	51.5	0	0	27	0	0	0	23.2	0	0	8.8	2.9	16.4	16	2.6	4	37	67.7	264.8
Total Use	423.3	268.5	16.6	108.3	53.2	26.9	97.2	23.8	497.2	162.4	139.9	62.4	48.6	76.1	18	45	59.2	16.6	9.8	155.7	130.6	2439.3

Note: Coil (crude oil), Oilp: Oil products), RefG (Refinery gas), GasT (Gasoline for transportation), FGas (jet fuel), FGasBunk (Jet fuel bunkered), DieT (Diesel for transportation), DietTBunk (Diesel for transportation - bunkered), NGasExt (Natural gas extraction), NGasCons (Natural gas consumption – incl. city gas), CC (coal and smoke), Waste (Waste), RE (Renewable energy), Straw (Straw), FW (Firewood and wood chips), WP (wood pellets), BioG (Biogas), BBB (Biodiesel, bioethanol and bio-oil), El (electricity), DHeat (District heat).

For each of the 21 energy types, we have both supply and usage data for each industry, represented by the first nine rows in each block. Specifically for energy supply, energy can be imported or produced using accessible inputs in the form of waste or renewable energy (e.g., wind or solar power). In the usage block, energy can be exported for use outside Denmark, consumed by households, or lost due to distribution losses. If there is an excess supply (usage) of a specific energy type, this will increase (lower) the inventories of that energy type (see the row "Change in inventories"). As a result, total energy usage equals total energy supply for each industry.

Although we have both the supply and usage data at an industrial level, in most cases, we cannot see what fraction of the total energy used by an economic unit is supplied by a specific supplier. We can only establish partial links based on our abductive reasoning of the energy accounts, which could be useful in interpreting the results. For instance, from a consumption point of view, the 'energy producing and refineries' industry shows a high consumption of crude oil (328.7 million gigajoules), indicating its substantial reliance on the mining industry. Whereas, from a production point of view, this industry serves as the sole provider of natural gas for consumption and the biggest provider of district heating, the latter being the largest source of energy for the household sector used in heating residential units. Thus, the household sector, for its energy consumption, appear to have a strong direct dependence on energy producing and refineries industry.

Unfortunately, we do not have the full information to transparently link each and every unit of energy between suppliers and users. This limitation, along with the absence of energy prices required to convert physical units to monetary units, prevents us from integrating energy accounts in the IO setup presented earlier. Nonetheless, it is important to emphasise that the costs (revenue) associated with using (selling) energy as an intermediate good are implicit in the IO table. In this version of the model, we simply aim to establish a relationship between energy usage (in physical units) and economic activity while capturing the resulting GHG emissions. A more realistic extension of the model will require accounting for the price of each energy type, converting physical units to monetary units, and then integrating energy accounts in the IO setup.

4.3.2. Emission Accounts

We collect data on six types of emissions: carbon dioxide (CO2), nitrous oxide (N2O), methane (CH4), sulfur hexafluoride (SF6), perfluorocarbons (PFC), and hydrofluorocarbons (HFC). These emissions are later used to calculate the CO2-equivalent measure (CO2E). As mentioned, we use a mix of data sources for the emissions data. For CO2, N2O, and CH4, we use data from the GreenREFORM model databank (Svarer et al., 2024). In contrast, we use data from Statistics Denmark for SF6, PFC, and HFC. The choice of data source depends on how the emission type is produced. Since CO2, N2O, and CH4 are emitted both in relation to energy usage and independently of it, the detailed setup of the GreenREFORM model databank allows us to disaggregate energy-related emissions down to specific energy types. In the table below, we present this disaggregation for CO2 emissions related to energy: [17]

Table 6: CO2 emissions related to energy usage

CO2 Emissions accounts 2017	Coil	Oilp	RefG	GasT	Fgas	FGasBunk	DieT	DieTBunk	SGasBunk	NGasExt	NGasCons	CC	Waste	RE	Straw	FW	WP	BioG	BBB	El	Dheat	Total
Agriculture	0	911.95	0	29.96	0	0	167.03	0	0	0	98.37	38.04	19.88	0	198.29	3	0	15.57	12.47	0	0	1494.56
Foresty	0	26.6	0	1.8	0	0	33.83	0	0	0	0	0	0	0	0	0	0	0	2.53	0	0	64.76
Fishery	0	361.76	0	0.87	0	0	6.72	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	369.85
Mining	0	44.66	0	0.2	0	0	8.14	0	0	1312.05	50.72	0	1.26	0	0	0.42	113.72	0	0.61	0	0	1531.78
Manufacturing of food products	0	163.25	0	2.89	0	0	70.55	0	0	0	670.11	138.99	74.1	0	0	24.55	0.53	52.92	5.27	0	0	1203.16
Energy production and refineries	0	216.59	909.64	8.2	0	0	29.82	0	0	0	3053.88	5724.74	877.5	0	1525.44	2616.56	4485.71	1083.33	2.23	0	0	20533.64
Energy intensive industries	0	845.22	0	5.55	0	0	184.15	0	0	0	754.63	287.73	3980.75	0	0	103.99	21.76	36.74	13.75	0	0	6234.27
Final corporations	0	5.1	0	7.01	0	0	29.1	0	0	0	10.07	0	0	0	0	0	0	0.65	2.17	0	0	54.1
Other industries	0	1673.88	0	362.22	1367.88	2208.82	4562.32	995.78	35259.76	0	592.62	0	68.91	0	0	39.38	155.98	63.32	340.68	0	0	47691.55
Households	0	655.72	0	3776.09	0	0	2038	0	0	0	1423.13	0	0	0	297.44	2820.77	1637.7	90.78	152.18	0	0	12891.81
Total CO2 (1000 tons)	0	4904.73	909.64	4194.79	1367.88	2208.82	7129.66	995.78	35259.76	1312.05	6653.53	6189.5	5022.4	0	2021.17	5608.67	6415.4	1343.31	532.39	0	0	92069.48

[17] This data was published together with the presentation of the green reform model used to evaluate environmental regulations of the agricultural industry in Denmark (Svarer et al. 2024). Duo to inconsistency with energy used and emissions for wood pills for the agricultural industry, we have modified the databank by setting emission for the agricultural sector associated with wood pellets to 0. This only occurs in 1995, 2000, and 2003 and the highest absolute value being 4.37E-13.

As detailed emissions data from the GreenREFORM database are only available up until 2017, we construct new emissions data for 2018 and 2019, assuming the same relationship between energy usage and emissions as in 2017 (since energy data is available for 2018 and 2019). This will become clearer in section 5.4.2, where energy-specific emissions coefficients are calculated.

For SF6, PFC, and HFC emissions, which are unrelated to energy usage, it is neither necessary nor possible to disaggregate these emissions into the format of Table 6. This data is available for the entire time period, and no further operations are required.

This concludes the data requirements for the model, which relies on national accounting data at both the industry and sectoral levels, along with emissions and energy data. In the next section, we introduce the structure of the model.

5. The model structure

The model has four major blocks, i) the domestic production block, describing total production, production costs, and profits for the domestic industries, ii) the aggregate (or final) demand block, providing a detailed description of the drivers of aggregate demand. Since the model is demand-driven, this block to a large degree determines industrial production (or the supply side), which is presented in the production block. iii) the stock-flow consistency block, in which we model the financial aspects of each institutional sector to capture the interdependence between real and financial spheres of the economy, while ensuring there are no leakages in the system, and iv) the environmental block, where we model the types of energies used in the production process and also capture the resulting GHG emissions of each industry and the economy as a whole.

5.1. Production components

5.1.1. Production

We can formally show the accounting identity used to calculate total production. As mentioned in section 4.1.2, we use a dynamic setup where we rely on a single equation (see equation 4), according to which, total domestic production in fixed prices is defined as follows:

$$prod_{dom,t}^{n} = \sum_{i=1}^{9} z_{dom,t}^{n\,i} + \sum_{p=1}^{7} c_{dom,t}^{n\,p} + gov_{dom,t}^{n} + inv_{dom,t}^{n} + \Delta invent_{dom,t}^{n} + x_{dom,t}^{n} \text{(equation 6a)}$$

In the above equation, $\sum_{i=1}^{9} z_{dom,t}^{n\,i}$ represents the sum of inputs (or total intermediate goods) sold by the domestic industry *n* to the domestic industry *i* (where *i=1,2,3,…,9*). That is, it represents the sum across columns (representing sales) while holding the row *fixed* for an industry *n*. For example, if we take the row that represents the agricultural sector, the sum of column 1 (left) to column 9 (right) will represent the total intermediate goods sold by the agricultural sector. The term $\sum_{p=1}^{7} C_{dom,t}^{n\,p}$ in the equation represents consumption of different types of goods (*p*) offered by industry *n*. In other words, this represents the sum across the columns (representing sales of different good types) in the household consumption block while holding the row fixed for an industry *n*. The remaining elements of equation 6a, given by $gov_{dom,t}^{n}$, $inv_{dom,t}^{n}$, $\Delta invent_{dom,t}^{n}$, and $x_{dom,t}^{n}$ represent the production of industry *n* (everything in deflated values with base year 2010) for the purpose of public spending, investment, changes in inventories, and exports, respectively.[18] By including equation 5d ($z_{t}^{i\,n} = a_{t}^{i\,n} * prod_{dom,t}^{n}$) in the model we take into account the indirect effect of changes in real production on required inputs. It is important to highlight that this relationship is modelled using the deflated variables. If this relationship was captured using nominal values, an increase in prices for final consumption goods in industry *n* would result in an increase in inputs from other industries ($\sum_{i=1}^{9} Z_{dom,t}^{n\,i}$). We can now compute total production ($PROD_{dom,t}^{n}$) in nominal terms, using the price indices in Table 3 as follows.

$$PROD_{dom,t}^{n} = \sum_{i=1}^{9} Z_{dom,t}^{n\,i} + \sum_{p=1}^{7} C_{dom,t}^{n\,p} + GOV_{dom,t}^{n} + INV_{dom,t}^{n} + \Delta INVENT_{dom,t}^{n} + X_{dom,t}^{n} \quad \text{(equation 7a)}$$

where $Z_{dom,t}^{n\,i} = z_{dom,t}^{n\,i} * py_{t}^{n}$ is nominal intermediate goods sold by industry *n* to other industries, $\sum_{p=1}^{7} C_{dom}^{n\,p} = \sum_{p=1}^{7} c_{dom}^{n\,p} * py_{t}^{n}$ is nominal private consumption of goods, $GOV_{dom,t}^{n} = gov_{dom,t}^{n} * py_{t}^{n}$ is nominal government consumption of goods produced by a domestic industry *n*, $INV_{dom,t}^{n} = inv_{dom,t}^{n} * py_{t}^{n}$ is nominal investment using goods of a domestic industry *n*, $\Delta INVENT_{dom,t}^{n} = \Delta invent_{dom,t}^{n} * py_{t}^{n}$ is the nominal change in inventories, and $X_{dom,t}^{n} = x_{dom,t}^{n} * py_{t}^{n}$ is the nominal exports of goods by a domestic industry *n.*

[18] From total production, we can derive the sales within each industry by deducting the change in inventories from the production $sale_{dom,t}^{n} = prod_{dom,t}^{n} - \Delta invent_{dom,t}^{n}$. If industry *n* produces more than what is sold, the change in inventories will be positive, whereas a negative value indicates that goods produced in previous years were sold in the current period, lowering the value if the inventories. Since changes in inventories are relatively small and treated as exogenous, we do not distinguish between sales and production.

5.1.2. Cost of production

To estimate the costs of a specific domestic industry, we can move vertically down the IO table (see Table 2), where all the entries, except gross operating surplus and mixed income, represent the costs associated with production. The total cost of production ($COST^{n}_{dom}$) for a domestic industry *n* in nominal terms is determined as follows:

$$COST^{n}_{dom,t} = \sum_{i=1}^{9} Z_t^{i\,n} + Z^{n}_{uim,t} + Z^{n}_{imd,t} + CT^{n}_t + VAT^{n}_t + OPT^{n}_t + W^{n}_t \qquad \text{(equation 8)}$$

where $\sum_{i=1}^{9} Z_t^{i\,n}$ represents total inputs purchased by industry *n* (incl. both domestic and imported inputs). That is, each type of input (e.g., agricultural input) can be divided into domestic and imported input as follows:

$$Z_t^{i\,n} = Z^{i\,n}_{dom,t} + Z^{i\,n}_{im,t} \qquad \text{(equation 9a)}$$

$$\sum_{i=1}^{9} Z_t^{i\,n} = \sum_{i=1}^{9} Z^{i\,n}_{dom,t} + \sum_{i=1}^{9} Z^{i\,n}_{im,t} \qquad \text{(equation 9b)}$$

Here, $\sum_{i=1}^{9} Z^{i\,n}_{dom,t}$ is the cost of domestic inputs purchased by industry *n* from domestic industry *i* at time *t*. The term $\sum_{i=1}^{9} Z^{i\,n}_{im,t}$ denotes the sum of inputs purchased by industry *n* from foreign industries at time *t*. Note that the notation $Z_t^{i\,n}$ is the sum across rows *i* (where *i = 1,2,3,…,9*) while holding a column *n* fixed, which gives us inputs as *costs* for an industry *n.* For example, to calculate total domestic inputs purchased by the agricultural sector, we choose the column, representing the agricultural sector, and then take sum of row 1 (top) to row 9 (bottom*)*. Note that the notation $Z_t^{i\,n}$ in equation 8 is different from the notation $Z_t^{n\,i}$ denoting inputs as *sales* in equation 7 (which was the sum across columns *i* while holding a row *n* fixed). In the rest of the presentation*,* $Z_t^{i\,n}$ will represent costs of inputs for an industry *n* whereas $Z_t^{n\,i}$ will represent sales of an industry *n,* in both cases with industry *i* as the counterpart.

For the remaining notations in equation 8, $Z^{n}_{uimp,t}$ denotes the unspecified imports used, $Z^{n}_{imd,t}$ represents the import duties paid, CT^{n}_t denotes the commodity taxes, VAT^{n}_t denotes the value added taxes, OPT^{n}_tdenotes other production taxes, and W^{n}_t denotes the wage bill paid by industry *n*.

The costs related to $Z^{n}_{uimp,t}$ and $Z^{n}_{imd,t}$ will be addressed in section 5.2.6, where we discuss imports, while the remaining costs will be covered now as we begin modeling the various taxes associated with production. In this context, commodity taxes, value-added taxes, and other production taxes are calculated using an exogenous time-varying rate for each industry. For commodity taxes in each industry (CT_t^n), we multiply domestically purchased inputs by the corresponding tax rate as follows:

$$CT_t^n = \left(\sum_{i=1}^{9} Z_{dom,t}^{i\,n} \right) * CT_{rate,t}^n \qquad \text{(equation 10)}$$

Using the same strategy, we calculate value added taxes (VAT_t^n) by multiplying the domestically purchased inputs with the corresponding tax rate as follows:

$$VAT_t^n = \left(\sum_{i=1}^{9} Z_{dom,t}^{i\,n} \right) * VAT_{rate,t}^n \qquad \text{(equation 11)}$$

Regarding other production taxes paid by the 9 industries (OTP_t^n), we distinguish between environmental taxes ($ENV_{tax,t}^n$) and non-environmental taxes ($NE_{tax,t}^n$). The identity representing total other production taxes can be represented as follows:

$$OTP_{tax,t}^n = \ NE_{tax,t}^n + ENV_{tax,t}^n \qquad \text{(equation 12)}$$

To compute non-environmental (net) other production taxes, we use an exogenous rate based on total inputs used from domestic production. This can be expressed as follows:

$$NE_{tax,t}^n = \left(\sum_{i=1}^{9} Z_{dom,t}^{i\,n} \right) * NE_{rate,t}^n \qquad \text{(equation 13)}$$

The environmental taxes ($ENV_{tax,t}^n$) are determined endogenously in the model and will be further discussed in section 5.4.3 (equation 144a).

The final component of industry specific costs is the wage bill W_t^n paid to workers by the nine industries. Here we multiply the wage rate $Wage_t^n$ with the number of employees in each industry:

$$W_t^n = Wage_t^n * EMP_t^n \qquad \text{(equation 14)}$$

If the total nominal value of production (in equation 7) of an industry are higher than its nominal costs (in equation 8), the industry will have a positive mixed income and gross operating surplus (or profits), which is represented in the last row of the value-added block. This identity can be represented as follows:

$$PROFIT_t^n = PROD_{dom,t}^n - COST_{dom,t}^n \qquad \text{(equation 15)}$$

where $PROFIT_t^n$ in equation 15 represents the profit (gross operating surplus and mixed income) of industry *n*. For example, the gross operating surplus and mixed income for the agricultural industry is 21.5 billion DKK in 2019 (see Table 2), which is equivalent to the difference between total production measured as the sum of the first row and the total costs measured by the sum of the first column (excluding the gross operating surplus and mixed income itself).

The calculations of profits as a residual ensures that we have a full account of production in the IO framework where total output of a given industry *n* equal its total outlays (e.g., total output and outlays for agriculture sector are 77.232).

5.1.3. The labour market: Prices, wages, and employment

For price setting at an industry level, we follow a standard pricing mechanism often used within the Post-Keynsian literature (Harris 1974, Asimakopulos 1975, Godley and Lavoie 2006). We assume that producers have some degree of market power (or monopoly) and can therefore set prices above the unit cost of production. This approach to price setting is realistic when applied at an industrial level, since products across the industries are not close substitutes. To estimate the price within each industry, we assume the price for a product offered by industry *n* is a function of a mark-up over unit cost of production as follows:

$$py_t^n = (1 + \mu_t^n) * \frac{COST_{dom,t}^n}{prod_{dom,t}^n} \qquad \text{(equation 16)}$$

where μ_t^n is a markup, $COST_{dom,t}^n$ is the total nominal cost of production (see equation 8) and $prod_{dom,t}^n$ is the total (real) production (see equation 6). The mark-up differs across industries as well as over time, and it is determined exogenously.[19] Moreover, our price setting in equation 16 implies that consumption prices are also endogenous, since consumer prices to a large degree depend on producer prices py_t^n (for more information on the consumer prices see appendix 8.4).

It should be re-emphasised that, from a demand perspective, there is no direct substitution effect between the industries. However, substitution between industries can occur indirectly, as we include substitution between i) the seven product types and ii) domestic and foreign goods. In the first case, a higher price for meat products will shift demand to other products (mainly other food products).

[19] The mark-up is calculated as: $\mu_t^n = \frac{(PROD_{dom,t}^n - COST_{dom,t}^n)}{COST_{dom,t}^n}$. The numerator represents nominal profits while the denominator shows total cost of production also in nominal values.

While in the second case, an increase in the price of domestic agricultural products relative to foreign agricultural products will increase the demand for foreign agricultural products (both used as inputs and in final consumption). For more information about these two substitution effects go to appendix 8.4.

We now define employment, which, like in most Post-Keynesian models, is determined by two fundamental factors; the level of economic activity and productivity. Applying this approach at an industrial level, employment level (or the number of employees) in each industry can be defined as a function of total production of industry *n* to the corresponding productivity ratio:

$$EMP_t^n = \frac{prod_{dom,t}^n}{a_t^n} \qquad \text{(equation 17a)}$$

Here $prod_{dom,t}^n$ is real total production for a given industry and a_t^n is the productivity within each industry. Productivity is assumed to be exogenous.[20]

Using the number of employed in each industry, we calculate the number of unemployed using the labour force:

$$UNEMP_t = LF_t - \sum_{n=1}^{9} EMP_t^n \qquad \text{(equation 17b)}$$

We can now calculate the unemployment rate as follows:

$$UR_t = \frac{UNEMP_t}{LF_t} \qquad \text{(equation 17c)}$$

[20] Productivity is calculated outside the model, given by the following equation: $prod_t^n = \frac{prod_{dom,t}^n}{EMP_t^n}$.

To model wages, we first estimate a general wage rate in the economy. The general wage rate is determined by a targeted general wage rate ($Wage_t^{gen,T}$) and the average level of productivity in the economy (a_t).[21] The estimated function form of this equation is as follows:

$$ln\Delta\left(Wage_t^{gen}\right) = 0.24^{***} + 0.78^{***} ln\Delta\left(Wage_{t-2}^{gen}\right) + 0.08^{**} ln\Delta\left(Wage_{t-2}^{gen,T}\right) - 0.15^{**}\ln\left(Wage_{t-1}^{gen}\right) + 0.09^{*}\ln\left(Wage_{t-2}^{gen,T}\right) + 0.28^{***}\ln(a_{t-1}) \qquad \text{(equation 18a)}$$

According to the estimates, productivity only affects the wage rate in the long-run whereas targeted wages can affect the wage rate both in the short run and the long run. Workers are assumed to set a targeted general wage rate while taking into account previous inflation; it is defined as the general wage last year times last year's rate of inflation:

$$Wage_t^{gen,T} = Wage_{t-1}^{gen} * (1 + \pi_{t-1}) \qquad \text{(equation 18b)}$$

Once we have estimated the general wage in the economy, we can endogenies the industry-specific wage rates to capture the heterogeneity in wages across the industries. The wage rate within each industry is a linear function of the general wage rate:

$$\ln\left(Wage_t^n\right) = \omega_0^n + \omega_1^n \ln\left(Wage_t^{gen}\right) \qquad \text{(equation 18c)}$$

In equation 18c, ω_0^n should be seen as an industry specific wage premium, for example if an industry relies on specialized workers the wage premium is expected to be above average in the economy. The coefficient ω_1^n determines the industry specific bargaining power, when the general wage level in the economy ($Wage_t^{gen}$) goes up a value of ω_1 below (above) 1 indicates that the industry has relatively low (high) bargaining power.

This concludes the production components within the model. The reader should note how all the production components presented in this section are functions of either nominal or real production (see equation 5c). As production is driven by final demand, this means that the model is demand driven, in which production components adjust to final demand.

[21] The average level of productivity is exogenously calculated as the ratio between real GDP and total employment as follows: $a_t = \frac{y_t}{emp_t^{tot}}$

5.2. Aggregate demand components

In this section, we will cover aggregate demand components which include consumption, investments, changes in inventories, government spending, exports, and imports. When modeling consumption and investment, we follow a two-step strategy: i) we estimate the aggregate flow using regression equation. ii) Then, we split the estimated aggregate value into its corresponding sub-categories pertaining to industrial products. The share of each industrial product in the aggregate flow is then endogenized based on the relative price level between products using a nested structure. Following this approach, we capture the asymmetric response of industrial products to income and price shocks. Changes in inventories and government spending are exogenous in the model and will only be shortly described. When modeling the trade balance, we estimate the behavioural equations for each industry to capture the asymmetric responses across the industries following movements in domestic and foreign prices.

5.2.1. Consumption

When modeling consumption, we use the aggregate value of household consumption before taxes. This aggregate consumption (without value added taxes and commodity taxes) is denoted by c_t^{tot} to distinguish it from the definition of aggregate consumption C_t^{agg} presented in equation 55a.

Following standard post-Keynesian theory, we estimate consumption as a linear function of disposable income and wealth (see, e.g., Byrialsen and Raza 2022; Thomsen et al. 2024). This relationship is captured by the following Error Correction Model (ECM) model:

$$\ln\Delta(c_t^{tot}) = 0.37^{**}\, ln\Delta yd_t^H - 0.30^{***}\, ln\, c_{t-1}^{tot} + 0.29^{***} \ln yd_{t-1}^H + 0.01 \ln fnw_{t-1}^H - 0.002^{**}\, Trend \quad \text{(equation 19)}$$

Real consumption (c_t^{tot}) in the short run is determined by real disposable income (yd_t^H), while in the long run, it is determined by real disposable income and real financial net wealth (fnw_{t-1}^H).[22] The estimates are in line with our expectation, suggesting positive effects of disposable income and financial net wealth on consumption.

After estimating consumption, we split the aggregate value into its corresponding sub-categories; the consumption basket consists of 7 types of consumption goods, bread (c_t^{110}), meat (c_t^{120}), fish (c_t^{130}), dairy (c_t^{140}), fruits and vegetables (c_t^{160}), other food products (c_t^{180}), and finally industry

[22] To calculate real disposable income (yd_t^H), and real financial net wealth (fnw_t^H) we use the tax-adjusted consumer price index calculated in appendix 8.4.

specific products (c_t^{spec}). The industry specific products are simply the type of products (other than the bread, meat, fish, dairy, and fruits & vegetable) provided by each industry *n*. For instance, energy production and refinery industry, does not supply any of these food products, so its main supply – consisting of different energy types – is captured in the industry specific flow. The superscripts are solely for identification purposes, matching the notation used by Statistics Denmark. For a general representation, we replace the superscript by p (where *p = 1,2, 3, 4, 5, 6, 7*) representing the 7 product types. The amount spent on a good type (*p*) is calculated as:

$$c_t^p = \gamma_t^{c^p} * c_t^{tot} \qquad \text{(equation 20a)}$$

Where $\gamma_t^{c^p}$ is the (time varying) fraction of a good type (*p*) in the total consumption; we endogenize this share using a nested structure with constant elasticity of substitution based on relative prices. First, consumers choose between food products and industry specific products, afterwards they make a choice amongst the 6 food products. In appendix 8.4, we show how each of the seven shares are calculated following the nested structure.

Once we find the amount spent by households on the consumption of good type p (e.g., meat), we can calculate what proportion of this good is provided by domestic producers and what proportion is provided by foreign producers.[23] We can calculate the value of domestic products ($c_{dom,t}^p$) and imported products ($c_{im,t}^p$) used in consumption as follows.

$$c_{dom,t}^p = (1 - \phi_{c,t}^p) * c_t^p \qquad \text{(equation 20b)}$$

$$c_{im,t}^p = (\phi_{c,t}^p) * c_t^p \qquad \text{(equation 20c)}$$

where $\left(1 - \phi_{c,t}^p\right)$ is the time varying fraction of the good produced by domestic producers whereas $\phi_{c,t}^p$ is the fraction of the good provided by foreign producers, i.e., it is the proportion that is imported.[24] After determining the share of domestic and foreign producers of a good type (*p*), we then disaggregate domestic and foreign production into the corresponding *n* industries (where *n=1,2,3,…,9*) according to Table 2. We use time-varying shares as follows:

$$c_{dom,t}^{n\,p} = \lambda_{dom,t}^{n\,p} * c_{dom,t}^p \qquad \text{(equation 21a)}$$

$$c_{im,t}^{n\,p} = \lambda_{im,t}^{n\,p} * c_{im,t}^p \qquad \text{(equation 21b)}$$

[23] This distinction is crucial to capture the effects of real exchange rate on aggregate demand.
[24] Later, in equation 43 we endogenize this fraction, allowing for substitution between domestically and foreign produced products.

where $c^{p,n}_{dom,t}$ is the amount of consumption of a good type (p) provided by the domestic industry n whereas $c^{p,n}_{im,t}$ is the amount imported from abroad, i.e., it is provided by foreign industry n.[25] Here, $\lambda^{p,n}_{dom,t}$ and $\lambda^{p,n}_{im,t}$ represent the exogenous share of each domestic and foreign industry in the supply of good (p).

We can now define nominal consumption (both for domestic and imported products) using price indices for each domestic (py^n_t) and foreign (pm^n_t) industry:

$$C^{n\,p}_{dom,t} = c^{n\,p}_{dom,t} * py^n_t \qquad \text{(equation 22a)}$$

$$C^{n\,p}_{im,t} = c^{n\,p}_{im,t} * pm^n_t \qquad \text{(equation 22b)}$$

We can now show the aggregate consumption across product types (p) and industries (n) (using both real and nominal values). First, we aggregate consumption across industries and calculate total nominal consumption for each product type (p):

$$C^{tot,p}_{dom,t} = \sum_{n=1}^{9} C^{n\,p}_{dom,t} \qquad \text{(equation 23a)}$$

$$C^{tot,p}_{im,t} = \sum_{n=1}^{9} C^{n\,p}_{im,t} \qquad \text{(equation 23b)}$$

$C^{tot,p}_{dom,t}$ represent *total* nominal consumption of product type (p) produced domestically (i.e., by all producers collectively) whereas $C^{tot,p}_{im,t}$ is the *total* nominal import of product type (p). We use the same strategy to calculate *total* real consumption of product type (p) produced domestically $c^{tot,p}_{dom,t}$ as well as the *total* real import $c^{tot,p}_{im,t}$:

$$c^{tot,p}_{dom,t} = \sum_{n=1}^{9} c^{n\,p}_{dom,t} \qquad \text{(equation 24c)}$$

$$c^{tot,p}_{im,t} = \sum_{n=1}^{9} c^{n\,p}_{im,t} \qquad \text{(equation 24d)}$$

[25] A share of imported consumption cannot be associated with an industry whereas these are classified as unspecified imports. We calculate unspecified imports as follows: $c^{p}_{uim,t} = \lambda^{p}_{uim,t} * c^{p}_{im,t}$. A similar equation is used for the other final demand components where some imports are also unspecified.

Next, we do the aggregation across product types *p* both in terms of nominal and real quantities. Since, we have 7 product types, we can express this set of equations as follows:

$$C_{dom,t}^{tot,n} = \sum_{p=1}^{7} C_{dom,t}^{n\,p} \qquad \text{(equation 25a)}$$

$$C_{im,t}^{tot,n} = \sum_{p=1}^{7} C_{im,t}^{n\,p} \qquad \text{(equation 25b)}$$

$$c_{dom,t}^{tot,n} = \sum_{p=1}^{7} c_{dom,t}^{n\,p} \qquad \text{(equation 25c)}$$

$$c_{im,t}^{tot,n} = \sum_{p=1}^{7} c_{im,t}^{n\,p} \qquad \text{(equation 25d)}$$

In the above set of equations, $C_{dom,t}^{tot,n}$ denotes the *total* nominal consumption of all 7 products produced by a domestic industry *n* whereas $C_{im,t}^{tot,n}$ is the *total* nominal imports of all 7 products produced by a foreign industry *n*. The equations with notations in small letters represent real quantities. As a final step, we can calculate total consumption for domestic and imported products by taking the sum across industries (both nominal and real).[26]

$$c_{dom,t}^{tot} = \sum_{n=1}^{9} c_{dom,t}^{tot,n} \qquad \text{(equation 26a)}$$

$$c_{im,t}^{tot} = \sum_{n=1}^{9} c_{im,t}^{tot,n} \qquad \text{(equation 26b)}$$

$$C_{dom,t}^{tot} = \sum_{n=1}^{9} C_{dom,t}^{tot,n} \qquad \text{(equation 26c)}$$

$$C_{im,t}^{tot} = \sum_{n=1}^{9} C_{im,t}^{tot,n} \qquad \text{(equation 26d)}$$

[26] It would have been equally correct to take the sum across product types.

Where $C_{dom,t}^{tot}$ represents *total* household consumption of goods produced by the domestic industry whereas $C_{im,t}^{tot}$ represents the *total* household consumption of imported goods. The share of imported goods in the Danish consumer basket is quite stable around 10 percent over the last decade.

To include taxes paid on final consumption we calculate exogenous tax rates based on the observed data. All final demand components are taxed through both commodity and value added taxes associated with each good type p:

$$C_{ctax,t}^{p} = rate_{ctax,t}^{cons,p} * C_{dom,t}^{tot,p} \qquad \text{(equation 27a)}$$

$$C_{VAT,t}^{p} = rate_{VAT,t}^{cons,p} * C_{dom,t}^{tot,p} \qquad \text{(equation 27b)}$$

The term $C_{dom,t}^{tot,p}$ in equation 27a and 27b represents consumption of the households for each good type p. Thus, $C_{ctax,t}^{p}$ is the commodity taxes and $C_{VAT,t}^{p}$ the value added taxes paid on the consumption of each good type p. To calculate total commodity taxes and value added taxes on consumption, we can take the sum across the good types in the consumption block as follows:

$$C_{ctax,t}^{tot} = \sum_{p=1}^{7} C_{ctax,t}^{p} \qquad \text{(equation 28a)}$$

$$C_{VAT,t}^{tot} = \sum_{p=1}^{7} C_{VAT,t}^{p} \qquad \text{(equation 28b)}$$

5.2.2. Investments

To estimate investment, we follow the approach of what is also known as the post-Kaleckian Bhaduri and Marglin (1990) model. According to this approach, investment is defined as a function of capacity utilization and profit share. The rate of capacity utilization is constructed as the ratio of real GDP (y_t) to the real stock of capital (k_t^{NFC}). Profit share (ps_t) is defined as the ratio of gross operating surplus to GDP. The estimated equation gives us the following:

$$\Delta \ln(inv_t^{tot}) = 1.8^{***} \Delta \ln\left(\frac{y_{t-1}}{k_{t-1}^{NFC}}\right) - 0.09^{***} \ln(inv_{t-1}^{tot}) + 1.42^{***}\, ps_{t-1} \qquad \text{(equation 29)}$$

According to the estimates, aggregate investment, in the short run, is driven by the rate of capacity utilization whereas in the long run, it is driven by the profit share. Note that our measure of total investments includes both domestic and imported goods used in capital formation (but excludes value-added taxes and commodity taxes for now).

We then use investment shares ($\lambda_{inv,t}^{n}$) to divide investment spending on the basis of industries, supplying goods used in investments.

$$inv_t^n = \lambda_{inv,t}^n * inv_t^{tot} \quad \text{(equation 30a)}$$

We can then divide investment in each industry into investment spending using domestic products and imported products. We use an import share ($\phi_{inv,t}^{n}$) calculated for each industry:

$$inv_{dom,t}^n = \left(1 - \phi_{inv,t}^n\right) * inv_t^n \quad \text{(equation 30b)}$$

$$inv_{im,t}^n = \left(\phi_{inv,t}^n\right) * inv_t^n \quad \text{(equation 30c)}$$

To convert real investments into nominal, we use the corresponding price deflators for domestic and foreign industries as follows:

$$INV_{dom,t}^n = inv_{dom,t}^n * py_t^n \quad \text{(equation 31a)}$$

$$INV_{im,t}^n = inv_{im,t}^n * pm_t^n \quad \text{(equation 31b)}$$

Lastly, we aggregate investment across industries for both nominal and real terms:

$$inv_{dom,t}^{tot} = \sum_{n=1}^{9} inv_{dom,t}^n \quad \text{(equation 32a)}$$

$$inv_{im,t}^{tot} = \sum_{n=1}^{9} inv_{im,t}^n \quad \text{(equation 32b)}$$

$$INV_{dom,t}^{tot} = \sum_{n=1}^{9} INV_{dom,t}^n \quad \text{(equation 32c)}$$

$$INV_{im,t}^{tot} = \sum_{n=1}^{9} INV_{im,t}^n \quad \text{(equation 32d)}$$

We can finally use exogenously calculated tax rates, to calculate the commodity taxes and value added taxes associated with domestically produced investment products.

$$INV_{ctax,t}^{tot} = rate_{ctax,t}^{inv} * INV_{dom,t}^{tot} \quad \text{(equation 33a)}$$

$$INV_{VAT,t}^{tot} = rate_{VAT,t}^{inv} * INV_{dom,t}^{tot} \quad \text{(equation 33b)}$$

5.2.3. Change in inventories

Changes in inventories are determined exogenously within the model. We choose to keep it exogenous in real values. The following equations describe real inventories, including domestic and $invent_{dom,t}^{tot}$ and import related $invent_{im,t}^{tot}$

$$\Delta invent_{dom,t}^{tot} = \sum_{n=1}^{9} \Delta invent_{dom,t}^{n} \quad \text{(equation 34a)}$$

$$\Delta invent_{im,t}^{tot} = \sum_{n=1}^{9} \Delta invent_{im,t}^{n} \quad \text{(equation 34b)}$$

Nominal inventories are partly endogenous in a sense that they are affected by (endogenous) domestic producer prices (py_t^n). Inventories in nominal terms can be represented as follows:

$$\Delta INVENT_{dom,t}^{n} = \Delta invent_{dom,t}^{n} * py_t^n \quad \text{(equation 35)}$$

As a result, the total change in inventories in nominal values are also calculated within the model:

$$\Delta INVENT_{dom,t}^{tot} = \sum_{n=1}^{9} \Delta INVENT_{dom,t}^{n} \quad \text{(equation 36)}$$

Finally, taxes pertaining to inventories are exogenous and are calculated within the model as follows:

$$\Delta INVENT_{ctax,t}^{tot} = rate_{ctax,t}^{invent} * \Delta INVENT_{dom,t}^{tot} \quad \text{(equation 37a)}$$

$$\Delta INVENT_{VAT,t}^{tot} = rate_{VAT,t}^{invent} * \Delta INVENT_{dom,t}^{tot} \quad \text{(equation 37b)}$$

5.2.4. Government spending

Government spending, like changes in inventories, is also treated as exogenous in real values. Final government consumption of imported ($gov_{im,t}^{tot}$) and domestic goods ($gov_{dom,t}^{tot}$) in real terms are computed as follows:

$$gov_{dom,t}^{tot} = \sum_{n=1}^{9} gov_{dom,t}^{n} \quad \text{(equation 38a)}$$

$$gov_{im,t}^{tot} = \sum_{n=1}^{9} gov_{im,t}^{n} \quad \text{(equation 38b)}$$

Again, as domestic prices can change in the model, so can the nominal value of government spending:

$$GOV_{dom,t}^{n} = gov_{dom,t}^{n} * py_{t}^{n} \qquad \text{(equation 39)}$$

We can then calculate the total government spending in nominal values as follows:

$$GOV_{dom,t}^{tot} = \sum_{n=1}^{9} GOV_{dom,t}^{n} \qquad \text{(equation 40)}$$

Finally, the commodity and value added taxes pertaining to government consumption are computed as follows:

$$GOV_{ctax,t}^{tot} = rate_{ctax,t}^{gov} * GOV_{dom,t}^{tot} \qquad \text{(equation 41a)}$$

$$GOV_{VAT,t}^{tot} = rate_{VAT,t}^{gov} * GOV_{dom,t}^{tot} \qquad \text{(equation 41b)}$$

For completeness, we have chosen to present equations related to the changes in inventories and government spending, even though they are not necessary for running the model. It is important for the reader to note that these two variables will only change in nominal terms and only for domestic industries.

5.2.5. Exports

We determine the total exports for each of the nine industries using a variant of the Armington model. Our exports in each industry are determined by price competitiveness and global demand. We can express the exports equation in log-linear form as follows:

$$\ln\left(\frac{x_t^n}{m_t^{n*}}\right) = \alpha_0^n + \alpha_1^n * \ln(rer_{t-1}^n) + adj_{x,t}^n \qquad \text{(equation 42)}$$

where $m_t^{n*}$is the global demand (or imports) for the type of goods produced by *n* type of industries across the globe. For example, when estimating the exports for the agricultural sector, x_t^n represents exports of Danish agricultural industry whereas m_t^{n*} represents the global demand for agricultural products. Thus, the left-hand side of equation 42 represents the share of Danish exports to total world imports within each specific industry. To calculate the share of Danish exports in the global market, we use the BACI-dataset providing import and export values amongst countries in the world at a product-level.[27] α_1^n captures the export elasticities to movements in real exchange rate

[27] As the BACI-dataset does not include products for all industries included by Denmark statistics, we use an average export share for the whole Danish economy based on all products in the BACI-dataset.

rer_t^n. Real exchange rate for each industry is a proxy for international competitiveness, defined as follows:

$$rer_t^n = xr_t \frac{py_t^n}{pm_t^n} \quad \text{(equation 43)}$$

where py_t^n is the domestic price of industry n, pm_t^n is the price of imports of industry n, both price indices are calculated in Danish currency whereas we do not need to consider the nominal exchange rate. To find the export elasticities (represented by α_1^n), we use estimates found by Kronborg, Poulsen, and Kastrup (2020) who derive export elasticities using the BACI-dataset for the Danish economy. As the BACI-dataset does not include exports for all industries, we use an average export elasticity for the Danish economy in the mining and financial corporation industries. The vector of the export elasticities pertaining to each industry is presented in the table below:[28]

Table 7: Export elasticities

Export elasticities (Industries)

	Statistics
Industry	**Export elasticities**
Agricultural	-5.13
Foresrty	-2.08
Fishery	-5.76
Mining	-1.90
Manufacturing food	-5.20
Energy supply and refinaries	-2.51
Energy intensive industries	-5.03
Financial corp	-1.90
Other industries	-3.95

Note:
Own calculations based on data from Svarer et al. (2024)

The negative values of the parameter(s) α_1^n imply that an increase (decrease) in the domestic price in industry n will appreciate (depreciate) the real exchange rate, which in turn will lower (raise)

[28] As we perform the aggregation to match the 9 industries in this paper, we use export within each industry to calculate a weighted average.

exports of industry n.[29] We set the constant (α_0^n) to match the logarithmic value of the first observation in the market share for each industry. Finally, we include an adjustment term ($adj_{x,t}^n$), which captures the effects of variables other than the real exchange rate. As the price elasticities presented by Kronborg, Poulsen, and Kastrup (2020) are based on micro data (with the goal of finding a causal relationship between relative prices and exports), these estimates are found to be theoretically intuitive, but poorly fitting the aggregate data. Leaving out the adjustment terms would therefore create large discrepancies compared to the observed data which could be problematic in the model simulations. An alternative approach is to obtain these elasticities using aggregate data. This approach, however, did not work in our case, as the estimates (using aggregate data) were found to be non-sensical due to small sample size. We believe it is crucial that the export elasticities have realistic signs and magnitudes, therefore, we make a strategic decision of using theoretically intuitive elasticities based on Danish micro data. After estimating the export share for each industry, we multiply the market share ($\frac{x_t^n}{m_t^{n*}}$) with the corresponding denominator (m_t^{n*}) to obtain exports of each industry in levels (x_t^n).

Once we have the total value of exports for each industry, we split this value into two categories, i) goods that are domestically produced and exported, and ii) goods that are first imported and then exported by the domestic industry. We use exogenous shares ($\phi_{x,t}^n$ and $1 - \phi_{x,t}^n$) based on the observed data to split exports:

$$x_{dom,t}^n = \left(1 - \phi_{x,t}^n\right) * x_t^n \qquad \text{(equation 44a)}$$

$$x_{m,t}^n = \left(\phi_{x,t}^n\right) * x_t^n \qquad \text{(equation 44b)}$$

where $x_{dom,t}^n$ represents goods that are domestically produced and exported, and $x_{m,t}^n$ represents goods that are first imported by industry *n* and then simply exported.

We can use the corresponding price deflators to convert exports from real into nominal values:

$$X_{dom,t}^n = x_{dom,t}^n * py_t^n \qquad \text{(equation 45a)}$$

$$X_{m,t}^n = x_{m,t}^n * pm_t^n \qquad \text{(equation 45b)}$$

To compute total exports for the Danish economy, we simply sum each type of exports across the industries as follows:

[29] We use elasticities whereas an export elasticity of -5.13 for the agriculture industry implies that a 1% increase in the real exchange rate for this industry lowers its export by 5.13%.

$$x^{tot}_{dom,t} = \sum_{n=1}^{9} x^{n}_{dom,t} \qquad \text{(equation 46a)}$$

$$x^{tot}_{im,t} = \sum_{n=1}^{9} x^{n}_{im,t} \qquad \text{(equation 46b)}$$

$$X^{tot}_{dom,t} = \sum_{n=1}^{9} X^{n}_{dom,t} \qquad \text{(equation 46c)}$$

$$X^{tot}_{im,t} = \sum_{n=1}^{9} X^{n}_{im,t} \qquad \text{(equation 46d)}$$

where $x^{tot}_{dom,t}$ denote total (real) exports of goods that are domestically produced and $x^{tot}_{im,t}$ denote exports of goods, which first enter the economy as imports.[30] In the above set of equations, the notations in capital letters represent nominal values.

Finally, we can calculate commodity and value added taxes associated with domestically produced nominal exports:

$$X^{tot}_{ctax,t} = rate^{x}_{ctax,t} * X^{tot}_{dom,t} \qquad \text{(equation 47a)}$$

$$X^{tot}_{VAT,t} = rate^{x}_{VAT,t} * X^{tot}_{dom,t} \qquad \text{(equation 47b)}$$

5.2.6. Imports

To model imports, we first estimate imports at an industrial level, followed by the estimation of imports in the final demand block. We start by determining the fraction of imports used as inputs in the industries. Recall that in equation 5d, we calculated total inputs using technical coefficients, these inputs consisted of both domestic and imported inputs used in production (also see equation 5c). To isolate the value of imported inputs, we use the share of imports ($\phi^{i\,n}_{z,t}$) in the total inputs and multiply it with total inputs ($z^{i\,n}_{t}$) as follows: [31]

$$z^{i\,n}_{im,t} = \phi^{i\,n}_{z,t} * z^{i\,n}_{t} \qquad \text{(equation 48a)}$$

[30] Note that an increase in exports as a result of real exchange rate depreciation will proportionately increase those imports which are linked to exports.
[31] Note that $z^{i\,n}_{t}$ is the real value of equation 2a.

$$z_{dom,t}^{i\,n} = \left(1 - \phi_{z,t}^{i\,n}\right) * z_t^{i\,n} \qquad \text{(equation 48b)}$$

where $z_{im,t}^{i\,n}$ is the value of imported inputs and $z_{dom,t}^{i\,n}$ is the value of inputs produced domestically. We can calculate these inputs in nominal terms by using the corresponding price deflators.

$$Z_{im,t}^{i\,n} = z_{im,t}^{i\,n} * pm_t^n \qquad \text{(equation 48c)}$$

$$Z_{dom,t}^{i\,n} = z_{dom,t}^{i\,n} * py_t^n \qquad \text{(equation 48d)}$$

From the IO data, we know that a fraction of imported inputs cannot be categorized using the Danish industry definitions - this is classified as unspecified imports. To determine unspecified imports, we assume a linear relationship between unspecified imports and total production of the domestic industry.

$$z_{uim,t}^n = \gamma_{uim,t}^n * prod_{dom,t}^n \qquad \text{(equation 49a)}$$

where $\gamma_{uim,t}^n$ is the (exogenous) share of unspecified imports in total production. We can also calculate the unspecified imports in nominal values as follows:

$$Z_{uim,t}^n = z_{uim,t}^n * pm_{uim} \qquad \text{(equation 49b)}$$

We now focus on endogenizing the share of imports $\phi_{z,t}^{i\,n}$ introduced in equation 48a-b. We assume that the share of imported inputs for each industry is partly driven by the real exchange rate. To determine import elasticities, we follow a simple strategy, known as the "rule of two" where the import elasticity of an industry is assumed to be half the export elasticity.[32] This approach is also adopted by the GreenREFORM model (Kirk and Hansen 2023). The main argument is that domestic residents, for a variety of reasons, have a preference for domestically produced goods even if they tend to be more expensive. The functional form of this relationship can be represented as follows:

$$\ln(\phi_{z,t}^{i\,n}) = \beta_0^{z^{i\,n}} + \beta_1^{z^n} * ln(rer_t^n) + adj_{\phi,t}^n \qquad \text{(equation 50)}$$

In the above equation, $\beta_0^{z^{i\,n}}$ is a constant taking the log of the starting value for the import share ($\phi_{z,t}^{i\,n}$), and $\beta_1^{z^n}$ represents the import elasticity (shown in table 8). We find that our computed elasticities are theoretically intuitive and within a justifiable range, when compared with a recent empirical study; Kastrup et al. (2023) use Danish industrial data to estimate import elasticities,

[32] In general, the rule of two is found to have strong empirical support, e.g., Feenstra et al. (2018) show that the "rule of two" cannot be rejected for almost 80% of all product types.

finding the macro elasticity to be 1.84, whereas the implied macro elasticity in our case (using weighted average of elasticities across the industries) is around 2.[33]

Finally, we have an exogenous determined adjustment term capturing other relevant effects than the real exchange rate $\left(adj_{\phi,t}^{n}\right)$.[34]

Table 8: Import elasticities (Industry)

Import elasticities (Industries)

	Statistics
Industry	**Import elasticities inputs**
Agricultural	2.50
Foresrty	1.00
Fishery	2.50
Mining	0.95
Manufacturing food	2.50
Energy supply and refinaries	1.20
Energy intensive industries	2.50
Financial corp	0.90
Other industries	2.00

Note:
Own calculations based on data from Svarer et al. (2024)

We now focus on imports associated with the final demand block. Focusing on household consumption, we first classify the consumer basket into domestic and imported goods. Since households' consumption basket consists of 7 types of goods, we carry out the classification for all 7 types. Recall the set of equations 20a-20b, where the proportion of good type (p) in the consumer basket imported from abroad was given by: $c_{im,t}^{p} = \left(\phi_{c,t}^{p}\right) * c_{t}^{p}$. Whereas the proportion of good type (p) in the consumer basket domestically produced was given by: $c_{dom,t}^{p} = \left(1 - \phi_{c,t}^{p}\right) * c_{t}^{p}$. Thus by endogenizing $\phi_{c,t}^{p}$, we can model the substitution effects between household consumption

[33] In general, these elasticities tend to vary a lot in the empirical literature. For example, Temere (2017) find that the estimated elasticities using micro data for Denmark ranges between -1.14 and -32.65 with an overall mean of -6.15 and a median of -4.45. Imbs and Mejean (2009) for the US data find elasticities across industries range from 3.1 to 28 with a standard deviation of 4.9.

[34] Like in the case of exports, the import elasticity is based on micro data estimations whereas the simulated import of inputs for each industry will fit the observed data poorly if the adjustment term is not included. We do this to not underestimate the import elasticities which is usually the case when using aggregate data (Kronborg, Poulsen, and Kastrup (2020)).

of domestic and foreign goods. To do so, we model $\phi_{c,t}^{p}$ as a function of the relative prices. We can represent this relationship as follows:

$$\ln(\phi_{c,t}^{p}) = \beta_0^{c^p} + \beta_1^{c^p} * \ln\,(rer_t^p) \qquad \text{(equation 51)}$$

The parameter $\beta_1^{c^p}$, estimated via OLS, captures the elasticity of substitution between domestic and imported goods.[35] The estimates of the import elasticities associated with the 7 product types are presented in table 9 where the highest elasticity is observed for Meat products.

Table 9 Import elasticities (Final consumption)

Import elasticities (Consumption)

	Statistics
Products	**Import elasticities final consumption**
Bread products	3.11
Meat products	3.94
Fish products	1.06
Dairy products	3.34
Fruit and vegetables	1.32
Other food products	2.11
Industry specific products	0.89

Note:
Own estimations based on data from Denmark Statistics

In equation 51, the real exchange rate rer_t^p is used as a proxy of competitiveness and is defined as follows:

$$rer_t^p = xr_t * \frac{ppcon_{dom,t}^{p}}{ppcon_{im,t}^{p}} \qquad \text{(equation 52)}$$

where $ppcon_{dom,t}^{p}$ is the domestic price and $ppcon_{im,t}^{p}$ is the import price of each good type p. Note that the definition of real exchange rate is different than the one presented in equation 43. More specifically, when modelling imports for consumers, we re-define the real exchange rate on the basis of the good type in the consumer basket. The construction of $ppcon_{dom,t}^{p}$ and $ppcon_{im,t}^{p}$ is explained in appendix 8.4.

[35] In contrast to equation 50 and 42 the import elasticities of final consumption presented in equation 51 are estimated using OLS on aggregated consumption data. As a result, the fitted values will be able to match the observed data and we do not include any adjustment term.

We can now define total imports associated with each demand component by accounting for unspecified imports. Total imports related to private consumption can be defined as the sum of specified and unspecified imports as follows:

$$M_{c,t}^{tot} = \sum_{n=1}^{9}\sum_{p=1}^{7} C_{im,t}^{n\,p} + \sum_{p=1}^{7} C_{uim,t}^{p} \qquad \text{(equation 53a)}$$

Where $\sum_{n=1}^{9}\sum_{p=1}^{7} C_{im}^{n\,p}$ captures the total value of imports in the consumption matrix by summing imports across the types of good p (represented by the column of the matrix) as well as across the type of industries (represented by the rows of the consumption matrix). Moreover, with each type of good, we also have unspecified imports, therefore we also include the sum of unspecified imports across the good types (since unspecified imports has nothing to do with industries and only varies across the goods type, we therefore take the sum across the good type p in this case).

By modeling the substitution between domestic and foreign goods for domestic industry and household consumption, we have modelled a large part of imports. The imports associated with the purpose of exports are partly endogenous in a sense that they increase proportionately with exports as explained in section 5.2.5. The remaining imports tied to spending on investment, inventories, and public consumption - which collectively constitute a small fraction - are not directly affected by the real exchange rate. Using the same approach, we can compute total imports associated with the remaining components of final demand. This can be expressed as follows:

$$M_{inv,t}^{tot} = \sum_{n=1}^{9} INV_{im,t}^{n} + INV_{uim,t} \qquad \text{(equation 53b)}$$

$$M_{invent,t}^{tot} = \sum_{n=1}^{9} \Delta INVENT_{im,t}^{n} + \Delta INVENT_{uim,t} \qquad \text{(equation 53c)}$$

$$M_{gov}^{tot} = \sum_{n=1}^{9} GOV_{im}^{n} + GOV_{uim,t} \qquad \text{(equation 53d)}$$

$$M_{x}^{tot} = \sum_{n=1}^{9} X_{m,t}^{n} + X_{uim,t} \qquad \text{(equation 53e)}$$

In the above set of equations, we have two components in each equation. The first component (e.g., $\sum_{n=1}^{9} GOV_{im}^{n}$) shows the sum of imported goods from 9 foreign industries. The second component in each of equation denotes unspecified imports associated with each final demand component. The last equation should be interpreted with caution; it represents the value of imports in the export block, representing goods, which first enter the economy as imports, but are later exported.[36]

Finally, both industries and final consumers pay import duties as they purchase foreign goods. We model the import duties using exogenous time-varying tax rates. For the 9 industries import duties are determined as follows: [37]

$$Z_{imd,t}^{n} = Z_{imd,t}^{rate,n} * \left(\sum_{i=1}^{9} \left(Z_{im,t}^{i\,n} \right) + Z_{uim,t}^{n} \right) \qquad \text{(equation 54a)}$$

And for the final demand components:

$$M_{cons,t}^{duty} = M_{duty,t}^{rate,c} * M_{C,t}^{tot} \qquad \text{(equation 54b)}$$

$$M_{gov,t}^{duty} = M_{duty,t}^{rate,g} * M_{GOV,t}^{tot} \qquad \text{(equation 54c)}$$

$$M_{inv,t}^{duty} = M_{duty,t}^{rate,inv} * M_{INV,t}^{tot} \qquad \text{(equation 54d)}$$

$$M_{invent,t}^{duty} = M_{duty,t}^{rate,invent} * M_{invent,t}^{tot} \qquad \text{(equation 54e)}$$

$$M_{x,t}^{duty} = M_{duty,t}^{rate,x} * M_{x,t}^{tot} \qquad \text{(equation 54f)}$$

The modelling of imports related to domestic final consumption and industry production implies that an increase in domestic income of households will increase imports via two channels, i) the direct channel, according to which, an increase in household income will induce consumption of imported goods, and ii) the indirect channel, according to which, an increase in households income will increase the demand for domestic goods, which in turn, will increase the imports of intermediate goods used in production of domestic goods.

5.2.7. Aggregating final demand

We now add some of the remaining elements to demand components to compute what we call the "aggregate" values, which are aggregate spending that matches the data published in the national accounts. These aggregate values are also used in our transaction flow matrix.

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[36] These re-exported imports usually never cross the Danish boarder.

[37] As industry level data for import duties are not available before year 2005, we use the total level of import duties and divide it out on industry level using fixed shares calculated using industry level data for year 2005.

To compute aggregate values, the spending on each aggregate demand component (excl. imports for now) equals the sum of consumption of domestic goods, consumption of imported goods, import duties, commodity taxes, and value added taxes as presented by the set of equations 55a-55e:

$$C_t^{agg} = C_{dom,t}^{tot} + M_{C,t}^{tot} + M_{cons,t}^{duty} + C_{ctax,t}^{tot} + C_{VAT,t}^{tot} \qquad \text{(equation 55a)}$$

$$GOV_t^{agg} = GOV_{dom,t}^{tot} + M_{GOV,t}^{tot} + M_{gov,t}^{duty} + GOV_{ctax,t}^{tot} + GOV_{VAT,t}^{tot} \qquad \text{(equation 55b)}$$

$$INV_t^{agg} = INV_{dom,t}^{tot} + M_{INV,t}^{tot} + M_{inv,t}^{duty} + INV_{ctax,t}^{tot} + INV_{VAT,t}^{tot} \qquad \text{(equation 55c)}$$

$$\Delta INVENT_t^{agg} = \Delta INVENT_{dom,t}^{tot} + M_{INVENT,t}^{tot} + M_{invent,t}^{duty} + \Delta INVENT_{ctax,t}^{tot} + \Delta INVENT_{VAT,t}^{tot} \qquad \text{(equation 55d)}$$

$$X_t^{agg} = X_{dom,t}^{tot} + M_{X,t}^{tot} + M_{x,t}^{duty} + X_{ctax,t}^{tot} + X_{VAT,t}^{tot} \qquad \text{(equation 55e)}$$

In the above set of equations, if we take aggregate consumption as an example, $C_{dom,t}^{tot}$ denotes total consumption of households for domestic products (see equation 26c), $M_{C,t}^{tot}$ denotes total specified and unspecified imports purchased by households (see equation 54b), $M_{cons,t}^{duty}$ are the import duties paid by households (see equation 54b), $C_{ctax,t}^{tot}$ and $C_{VAT,t}^{tot}$ are the total commodity taxes and total value added taxes paid by households, respectively (see equation 28b and 28b).

Finally, we can calculate aggregate imports by adding the relevant components as follows:

$$M_t^{agg} = \sum_{n=1}^{9}\sum_{p=1}^{7} Z_{im,t}^{n\,p} + \sum_{n=1}^{9} Z_{uim,t}^{n} + M_{C,t}^{tot} + M_{INVENT,t}^{tot} + M_{GOV,t}^{tot} + M_{INV,t}^{tot} + M_{X,t}^{tot} \qquad \text{(equation 55f)}$$

where $\sum_{i=1}^{9}\sum_{j=1}^{9} Z_{im,t}^{i\,n}$ is the sum of specified imported goods used in domestic production, and $\sum_{j=1}^{9} Z_{uim,t}^{n}$ is unspecified imports used in production. The remaining elements of the equation denoted by $M_{C,t}^{tot}, M_{GOV,t}^{tot}, M_{INV,t}^{tot}, M_{INVENT,t}^{tot}$, and $M_{X,t}^{tot}$ are the total imports associated with each of the final demand component as explained in equation 54a-54e.

We can write the GDP identity as follows:

$$Y_t = C_t^{agg} + GOV_t^{agg} + INV_t^{agg} + \Delta INVENT_t^{agg} + X_t^{agg} - M_t^{agg} \qquad \text{(equation 56)}$$

This completes the description of the supply and demand of goods in the economy.[38] We now move on to discussing the stock-flow-consistent block of the model, where we discuss financing decisions and budget constraints, and ensure the model has no leakages.

5.3. Stock-Flow-Consistency

In this section, we discuss the economy from a sectoral perspective, following the sectoral classification presented in the Transaction-Flow Matrix in Table 4. While discussing the financial aspects of each sector, we will outline the key steps and assumptions used in the process to ensure that the model achieves both vertical and horizontal consistency (Nikiforos and Zezza 2017), making it a stock-flow-consistent model. Until now, the economy has been disaggregated at an industrial level, but as we make the model Stock-Flow-Consistent, we will need to use a sectoral disaggregation of the national accounts.[39] As discussed earlier, industries are connected to the corresponding sectors when the gross operating surplus and mixed income in the TFM are described. In appendix 8.2, we provide an extended discussion on how the gross operating surplus and mixed income is transformed from an industry to sectoral level. We will now discuss the main characteristics of each institutional sector.

5.3.1. Non-Financial Corporations

We start by describing the gross operating surplus received by the NFC sector. To isolate the gross operating surplus received by the NFC sector, we distribute the industry level gross operating surplus ($PROFIT_t^n$) to the sectoral level ($B2_t^S$) using industry specific shares, based on the *industry by sector* matrix for GVA (described in appendix 8.2):

$$B2_t^{FC} = \sum_{n=1}^{9} PROFIT_t^n * S_{FC,t}^{profit,n} + \mathrm{adj}_t^{FC} \qquad \text{(equation 57a)}$$

$$B2_t^{H} = \sum_{n=1}^{9} PROFIT_t^n * S_{H,t}^{profit,n} + \mathrm{adj}_t^{H} \qquad \text{(equation 57b)}$$

$$B2_t^{G} = \sum_{n=1}^{9} PROFIT_t^n * S_{G,t}^{profit,n} + \mathrm{adj}_t^{G} \qquad \text{(equation 57c)}$$

[38] After calculating the aggregate variables in nominal values, we can use price indices taking the form of equation 21, to calculate $c_t^{agg}, gov_t^{agg}, inv_t^{agg}, invent_t^{agg}, x_t^{agg}$, and y_t.
[39] We make the transition from the industry to sectoral level as most variables required for making the model Stock-Flow-Consistent are only available at a sectoral level.

Equation 57a-57c is a weighted sum of the industry level profits where $PROFIT_t^n$ is the profit for each industry n multiplied by a share depending on the industries weight in a specific sector *s* ($S_{s,t}^{profit,n}$). We include an adjustment term for each sector to ensure a consistent transition from industry to sectoral level in which we match the observed data for both industry and sectoral gross operating surplus. The adjustment terms are necessary as the matrix of *industry by sector* is only available for 2016 and does not represent weights below 0.05 percent.[40]

In the above set of equations, we did not include profits for the non-financial corporations, which will be calculated as a residual. Note that the shares for each industry n sums to one ($S_{FC,t}^n + S_{H,t}^n + S_{G,t}^n + S_{NFC,t}^n = 1$).

$$B2^{NFC} = B2_t^{agg} - (B2^H + B2^{FC} + B2^G) \qquad \text{(equation 57d)}$$

Aggregate $B2_t^{agg}$ is calculated as the sum of profits at the industry level.

$$B2_t^{agg} = \sum_{n=1}^{9} PROFIT_t^n \qquad \text{(equation 57e)}$$

The transition from industry to sectoral level also needs to be performed for investments. Here, we follow a simple approach; total investment spending calculated from the industry level are split into sector specific investments using sector specific shares:[41]

$$I_t^{FC} = INV_t^{agg} * S_{FC,t}^{I,n} \qquad \text{(equation 58a)}$$

$$I_t^H = INV_t^{agg} * S_{H,t}^{I,n} \qquad \text{(equation 58b)}$$

$$I_t^G = INV_t^{agg} * S_{G,t}^{I,n} \qquad \text{(equation 58c)}$$

The investment of non-financial corporations (NFC) is then treated as a residual:

$$I_t^{NFC} = I_t^{agg} - (I_t^H + I_t^{FC} + I_t^G) \qquad \text{(equation 58d)}$$

Note that the investment data reported in the IO table represents production of investment goods by an industry, representing an inflow (the aggregate of which by definition equals domestic investment expenditures). Whereas the investments calculated in the set of equations 58a-58d are the investments expenditures by each sector, representing sectoral outflow.

[40] For more information see appendix 8.2.
[41] A similar approach is used for changes in inventories.

We can now calculate the net income received by the non-financial corporation sector as follows:

$$Y_t^{NFC} = Y_t - (B2_t^{agg} - B2_t^{NFC}) - \left(NTax_{prod,t} + \ OPT_t^{tot}\right) - W_t^{NFC} + NINT_t^{NFC} + NDIV_t^{NFC} + NOIR_t^{NFC} + NREFDI_t^{NFC} + OCT_t^{NFC} \qquad \text{(equation 59a)}$$

In the above equation, we can see that aggregate spending (forming GDP) is an inflow for the NFC, which is then adjusted for payments of production taxes ($NTax_{prod,t} + \ OPT_t^{tot}$) and the distribution of profits (i.e., the difference between aggregate profits and NFC profits equals the collective profits of other sectors as shown in 57d). We also subtract wage payments (W_t^{NFC}) which is the sum of wages paid to each industry (see equation 72a). We then add net income received on various financial assets taking the form of interest income on interest bearing assets ($NINT_t^{NFC}$), dividends ($NDIV_t^{NFC}$), income on other investments ($NOIR_t^{NFC}$), and retained earnings on FDI ($NREFDI_t^{NFC}$). Finally, we also account for net other current transfers received (OCT_t^{NFC}).

We can now calculate the disposable income of the non-financial corporations (YD_t^{NFC}) by subtracting income taxes paid by the non-financial corporations ($ITAX_t^{NFC}$) as follows:

$$YD_t^{NFC} = Y_t^{NFC} - ITAX_t^{NFC} \qquad \text{(equation 59b)}$$

Following the definition in the national accounts, the savings equation of non-financial firms (S_t^{NFC}) is represented as follows:

$$S_t^{NFC} = YD_t^{NFC} \qquad \text{(equation 60)}$$

NFC sector spends a part of its savings on investment (I_t^{NFC}), covering changes in inventories, acquisition and disposals of non-produced non-financial assets[42] (NP_t^{NFC}), and other capital transfers (CT_t^{NFC}). Therefore, we can subtract these flows from savings, and calculate net balance (or sectoral balance):

$$NL_t^{NFC} = S_t^{NFC} - I_t^{NFC} - \Delta INVENT_t^{NFC} - NP_t^{NFC} - CT_t^{NFC} \qquad \text{(equation 61)}$$

A positive (negative) net balance, reflecting a surplus (deficit), implies that the sector is spending less (more) than its income. We now focus on the vertical consistency of the TFM, for which we need to describe how the resultant surplus is spent or, in the case of a deficit, how it is financed. This requires specifying the financial aspects of the sector which includes the accumulation of financial assets.

[42] Non-produced non-financial assets refer to assets that have not been produced but have economic value and can be owned or exchanged. These assets include natural resources, contracts, licenses, etc., as well as other intangible assets.

As we specify how net lending (whether positive or negative) affects the accumulation of financial assets, we also describe the steps and assumptions used to ensure that sectoral balance (or net lending in equation 61) equates financial balance. In other words, we specify how net lending (whether positive or negative) affects the accumulation of financial assets. We start by defining the financial net lending (or financial balance) as follows:

$$FNL_t^{NFC} = NDEPO_{tr,t}^{NFC} + NSEC_{tr,t}^{NFC} + NLOA_{tr,t}^{NFC} + NEQ_{tr,t}^{NFC} + NINSU_{tr,t}^{NFC} + NDERV_{tr,t}^{NFC} + NTCRED_{tr,t}^{NFC} \qquad \text{(equation 62)}$$

where FNL_t^{NFC} denotes the financial balance, which is equal to the sum of (net) transactions associated with gold (gold is only relevant for the financial corporations and ROW), deposits ($NDEPO_{tr,t}^{NFC}$), securities ($NSEC_{tr,t}^{NFC}$), loans ($NLOA_{tr,t}^{NFC}$), equities ($NEQ_{tr,t}^{NFC}$), insurance/tech. reserves ($NINSU_{tr,t}^{NFC}$), financial derivatives ($NDERV_{tr,t}^{NFC}$), and trade credits ($NTCRED_{tr,t}^{NFC}$). Note that the net transaction linked to each financial stock is computed as the transaction related to the accumulation of a financial asset (which is an outflow for purchasing a financial asset) minus the transaction related to the liability of that asset (which is an inflow representing sources of funding).

To provide an overview of the balance sheet, we can calculate the financial net wealth (FNW_t^{NFC}) as follows:

$$FNW_t^{NFC} = NDEPO_t^{NFC} + NSEC_t^{NFC} + NLOA_t^{NFC} + NEQ_t^{NFC} + NINSU_t^{NFC} + NDERV_t^{NFC} + NTCRED_t^{NFC} \qquad \text{(equation 63)}$$

Financial net wealth is the sum of net deposits ($NDEPO_t^{NFC}$), securities ($NSEC_t^{NFC}$), loans ($NLOA_t^{NFC}$), equities (NEQ_t^{NFC}), insurance/tech. reserves ($NINSU_t^{NFC}$), financial derivatives ($NDERV_t^{NFC}$), and trade credits ($NTCRED_t^{NFC}$). Note that notations used for net financial *stocks* are distinguished from net financial *transactions* as the former include “tr” in the subscript.

At this point, one can proceed to modelling each financial stock included in equation 63. If there are *k* number of financial stocks, one approach is to model a maximum of *k-1* stocks and treat the last financial asset as a residual. For simplification, we choose to only model net loans (or business credit) of firms while treating other financial stocks as exogenous. The functional form of the equation determining the demand for loans is represented as follows:

$$\left(\frac{NLOA_t^{NFC}}{K_t^{NFC}}\right) = 0.26^{***} + 0.28^{**}\left(\frac{I_{t-1}^{NFC}}{S_{t-1}^{NFC}}\right) - 2.11^{***} r_{t-1}^{LOA} \qquad \text{(equation 64)}$$

equation 64 states that loan to capital ratio depends positively on investment to savings ratio $\left(\frac{I_t^{NFC}}{S_t^{NFC}}\right)$ and negatively on interest rate. The intuition is that investment in excess of savings, is partly financed through loans. Moreover, an increase in the cost of borrowing (represented by interest rate on loans) will lower the demand for loans.

After endogenizing the stock of loans, we can use the change in stock of loans to determine loan transactions using the accounting identity expressed as follows:

$$NLOA_{tr,t}^{NFC} = \Delta NLOA_t^{NFC} - NLOA_{rv,t}^{NFC} \qquad \text{(equation 65)}$$

where $NLOA_{rv,t}^{NFC}$ represents the stock revaluations (treated as exogenous), $\Delta NLOA_t^{NFC}$ the change in the stock of loans, and $NLOA_{tr,t}^{NFC}$ is the transaction of loans.

While treating other financial stocks as exogenous, we can describe the accounting identities to show their evolution over time. This is done by adding the net transactions and net re-evaluations to the last years stock of the net financial asset:

$$NSEC_t^{NFC} = NSEC_{t-1}^{NFC} + NSEC_{tr,t}^{NFC} + NSEC_{rv,t}^{NFC} \qquad \text{(equation 66)}$$

$$NINSU_t^{NFC} = NINSU_{t-1}^{NFC} + NINSU_{tr,t}^{NFC} + NINSU_{rv,t}^{NFC} \qquad \text{(equation 67)}$$

$$NDERV_t^{NFC} = NDERV_{t-1}^{NFC} + NDERV_{tr,t}^{NFC} + NDERV_{rv,t}^{NFC} \qquad \text{(equation 68)}$$

$$NTCRED_t^{NFC} = NTCRED_{t-1}^{NFC} + NTCRED_{tr,t}^{NFC} + NTCRED_{rv,t}^{NFC} \qquad \text{(equation 69)}$$

where *NSEC* represents net stock of securities, *NINSU* represents net stock of insurance and pensions, *NDERV* represents the net stock of derivatives, and *NTCRED* represents the net stock of trade credits. Note that notations for transactions and revaluations pertaining to each of the financial stock are distinguished by using subscripts *tr* and *rv*, respectively. In the above set of equations, for each financial stock, we can see that when current transactions and revaluations are added to the past value of a financial stock, we get the present value of the financial stock. In other words, changes in the value of a financial stock can occur for only two reasons, i.e., new transactions and revaluations.

We are now left with two financial stocks namely equities and deposits. Regarding equities, we assume that the stock market is demand-driven in a sense that new equities issued by non-financial corporations equal the demand for equities from other sectors. This relationship is captured in equation 70a as follows:

$$NEQ_{tr,t}^{NFC} = -\left(NEQ_{tr,t}^{FC} + NEQ_{tr,t}^{H} + NEQ_{tr,t}^{G} + NEQ_{tr,t}^{ROW}\right) \qquad \text{(equation 70a)}$$

where the transactions pertaining to the supply of equities by NFC is denoted by $NEQ_{tr,t}^{NFC}$, which is equal to the collective demand for equities by other sectors, given by the sum of the transactions of equities by households ($NEQ_{tr,t}^{H}$), financial corporations ($NEQ_{tr,t}^{FC}$), government ($NEQ_{tr,t}^{G}$), and rest of the world ($NEQ_{tr,t}^{ROW}$). This implies that the net stock of equities (representing a liability) for NFC equals the sum of the net stock of equities (representing assets) of other sectors:

$$NEQ_{t}^{NFC} = NEQ_{t-1}^{NFC} + NEQ_{tr,t}^{NFC} + NEQ_{rv,t}^{NFC} \qquad \text{(equation 70b)}$$

To ensure that sectoral balance (or net lending in equation 61) equates financial balance in equation 62, we treat one financial stock, namely deposits, as residuals, which in this case should be defined as follows:

$$NDEPO_{tr,t}^{NFC} = NL_{t}^{NFC} + NL_{adj,t}^{NFC} - (NSEC_{tr,t}^{NFC} + NLOA_{tr,t}^{NFC} + NINSU_{tr,t}^{NFC} + NDERV_{tr,t}^{NFC} + NTCRED_{tr,t}^{NFC} + NEQ_{tr,t}^{NFC}) \qquad \text{(equation 71a)}$$

Note that we include an extra adjustment term $NL_{adj,t}^{NFC}$ to account for the small discrepancy between financial net lending (FNL_{t}^{NFC}) and net lending (NL_{t}^{NFC}) present in the published data. Like other financial stocks, we treat stock revaluations of deposits as exogenous, and can show that the stock of deposits evolves according to equation 71b:

$$NDEPO_{t}^{NFC} = NDEPO_{t-1}^{NFC} + NDEPO_{tr,t}^{NFC} + NDEPO_{rv,t}^{NFC} \qquad \text{(equation 71b)}$$

This completes the description of how the balance sheet of NFC sector is related to the real side of the economy. While holding the rest of the financial stocks exogenous, we have modeled two main sources of funds: issuing equities and securing loans. We now proceed to discussing the income and financial aspects of the household sector.

5.3.2. Households

We start by describing the computation of income for the household sector. The major source of household income takes the form of labour income share (wages), paid by the NFC. First, we define the total amount of wages paid by the NFC sector as the sum of all wages paid by the nine industries:

$$W_{t}^{NFC} = \sum_{n=1}^{9} W_{t}^{n} \qquad \text{(equation 72a)}$$

We know that a small fraction of workers employed by the NFC sector is foreign labour. Therefore, we can subtract wages paid to non-residents from the total wages in equation 66a, the resultant of which will give us wages paid to domestic households. This calculation is carried out in equation 66b below:

$$W_t^H = W_t^{NFC} - W_t^{ROW} \qquad \text{(equation 72b)}$$

Where W_t^H represents wages received by the household sector and W_t^{ROW} denotes wages paid to foreign labour.

We can now calculate the net income received by households (Y_t^H) as follows:

$$Y_t^H = B2_t^H + W_t^H + NINT_t^H + NDIV_t^H + NOIR_t^H + NREFDI_t^H - SCON_{p,t}^H + SBEN_{r,t}^H + OCT_t^H \qquad \text{(equation 73a)}$$

Households income consists of various income types: gross operating surplus received by households is denoted by ($B2_t^H$), wages are denoted by (W_t^H), net income received on financial stocks ($NINT_t^H, NDIV_t^H, NOIR_t^H, NREFDI_t^H$), social benefits received ($SBEN_{r,t}^H$), and net other current transfers (OCT_t^H) and finally we deduct social contributions paid ($SCON_{p,t}^H$).

By subtracting income taxes from net income received by households (Y_t^H), we can calculate the disposable income for households:

$$YD_t^H = Y_t^H - ITAX_t^H \qquad \text{(equation 73b)}$$

We can now calculate household savings by subtracting aggregate private consumption (C_t^{agg}) and adjusting for pension savings (PEN_t^{adj}), which is a part of household savings (S_t^H), but are not accessible for consumption as they are placed in pension funds.

$$S_t^H = YD_t^H - C_t^{agg} + PEN_t^{adj} \qquad \text{(equation 74)}$$

We can then calculate the net lending (or sectoral balance) for the household sector:

$$NL_t^H = S_t^H - I_t^H - \Delta INVENT_t^H - NP_t^H - CT_t^H \qquad \text{(equation 75)}$$

where I_t^H denotes household investment, NP_t^H denotes transactions pertaining to the acquisition and disposal of non-produced non-financial assets (such as natural resources, contracts, licences, etc.), and CT_t^H denotes other capital transfers.

We now focus on modeling the financial side of households, where we roughly follow the same approach as for non-financial corporations. First, we define the equation for financial balance:

$$FNL_t^H = NDEPO_{t,tr}^H + NSEC_{t,tr}^H + NLOA_{t,tr}^H + NEQ_{t,tr}^H + NINSU_{t,tr}^H + NDERV_{t,tr}^H + NTCRED_{t,tr}^H \qquad \text{(equation 76)}$$

where FNL_t^H is the net lending, determined by net transactions pertaining to various financial assets. $NDEPO_{t,tr}^H$ denotes transaction related to deposits, $NSEC_{t,tr}^H$ denotes transactions related to securities, $NLOA_{t,tr}^H$ represents loan transactions, $NINSU_{t,tr}^H$ denotes transactions related to insurance and pensions, $NDERV_{t,tr}^H$ denotes transactions related to derivates, and $NTCRED_{t,tr}^H$ denotes transactions related to trade credits.

We now define the financial net wealth of the households as follows:

$$FNW_t^H = NDEPO_t^H + NSEC_t^H + NLOA_t^H + NEQ_t^H + NINSU_t^H + NDERV_t^H + NTCRED_t^H \qquad \text{(equation 77)}$$

FNW_t^H is the financial net wealth, determined by the net value of the financial stocks on the balance sheet of households.

For the household sector, we endogenies two financial stocks, which constitute a significant portion of the balance sheet. On the asset side, we model equities endogenously, whereas on the liability side, we model loans endogenously. To model equities, we first define a variable, equity to wealth ratio, representing the share of net equities to total financial wealth in the previous period.

$$EQ_{ratio,t}^H = \left(\frac{NEQ_t^H - NEQ_{rv,t}^H}{FNW_{t-1}^H}\right) \qquad \text{(equation 78a)}$$

Following the intuition of Tobins portfolio theory, which has now become a well-integrated part of the stock-flow consistent models, we assume that the share of equity in the financial wealth of households partly depends on the rate of return associated with equities.[43] The specific functional form of the estimated equation is represented as follows:

[43] Here we use a different rate of return compared to the dividend rate calculate in appendix 8.3. The rate used in equation 78b only focus on the household sector and furthermore includes the re-evaluations. For equities, the re-evaluations seem to be an important aspect in determining demand, whereas we include this in the rate of return.

$$\Delta EQ^{H}_{ratio,t} = 0.33^{*}\Delta EQ^{H}_{ratio,t-1} + 0.060\Delta\frac{\left(NDIV^{H}_{t-1} + NEQ^{H}_{rv,t-1}\right)}{NEQ^{H}_{t-2}} - 0.18EQ^{H}_{ratio,t-1} + 0.10^{*}\frac{\left(NDIV^{H}_{t-2} + NEQ^{H}_{rv,t-2}\right)}{NEQ^{H}_{t-3}} \quad \text{(equation 78b)}$$

The estimates imply that an increase in the return on equity will increase the share of equities in the wealth formulation. This is found to be the case both in the short-run and long-run. The intercept (0.063) can be interpreted as capturing the share of equities in wealth that is not dependent on the rate of return associated with equities. We can use the estimated share in equation 78b to calculate net equities held by the household as follows:

$$NEQ^{H}_{t} = EQ^{H}_{ratio,t} * FNW^{H}_{t-1} + NEQ^{H}_{rv,t} \quad \text{(equation 78c)}$$

The estimated stock value in equation 78c is then used to determine the transactions for equities (while treating the revaluations of equities as exogenous):

$$NEQ^{H}_{tr,t} = \Delta NEQ^{H}_{t} - NEQ^{H}_{rv,t} \quad \text{(equation 78d)}$$

To endogenies household loans, we first define the ratio of loans to disposable income.[44]

$$LOA^{H}_{ratio,t} = \left(-\frac{NLOA^{H}_{t}}{YD^{H}_{t}}\right) \quad \text{(equation 79a)}$$

We assume this ratio is positively influenced by investment and negatively influenced by the cost of the loan, represented by the interest rate on the loan. The estimated functional form of this relationship is given by equation 79b:

$$LOA^{H}_{ratio,t} = 0.91^{***}LOA_{ratio,t-1} + 2.28^{***}\left(\frac{I^{H}_{t}}{YD^{H}_{t}}\right) - 0.31r^{LOA}_{t} - 0.37^{***}D_{2016} \quad \text{(equation 79b)}$$

We can use the estimated share from equation 79b to calculate the stock of loans held by the household sector as follows:

$$NLOA^{H}_{t} = -LOA^{H}_{ratio,t} * YD^{H}_{t} \quad \text{(equation 79c)}$$

The stock value in equation 79c is then used to determine the transactions for loans (while treating the revaluations of loans as exogenous):

$$NLOA^{H}_{tr,t} = \Delta NLOA^{H}_{t} - NLOA^{H}_{rv,t} \quad \text{(equation 79d)}$$

[44] We multiply the ratio by -1 as households only have loans as liabilities, whereas the net loans are negative.

To ensure that sectoral balance of the household sector equates its financial balance, we again treat deposits, as residuals, which in this case should be defined as follows:

$$NDEPO_{tr,t}^{H} = NL_{t}^{H} + NL_{adj,t}^{H} - (NSEC_{tr,t}^{H} + NLOA_{tr,t}^{H} + NINSU_{tr,t}^{H} + NDERV_{tr,t}^{H} + NTCRED_{tr,t}^{H} + NEQ_{tr,t}^{H}) \qquad \text{(equation 80a)}$$

Here, again, we include an adjustment term $NL_{adj,t}^{H}$ to capture discrepancies between net lending and financial balance in the published statistics.

The transaction data on deposits can be used to determine the stock of deposits for the households ($NDEPO_{t}^{H}$) while treating revaluations as exogenous:

$$NDEPO_{t}^{H} = NDEPO_{t-1}^{H} + NDEPO_{tr,t}^{H} + NDEPO_{rv,t}^{H} \qquad \text{(equation 80b)}$$

The remaining financial assets on the balance sheet of the households are exogenously determined; these financial stocks evolve as follows:

$$NSEC_{t}^{H} = NSEC_{t-1}^{H} + NSEC_{tr,t}^{H} + NSEC_{rv,t}^{H} \qquad \text{(equation 81)}$$

$$NINSU_{t}^{H} = NINSU_{t-1}^{H} + NINSU_{tr,t}^{H} + NINSU_{rv,t}^{H} \qquad \text{(equation 82)}$$

$$NDERV_{t}^{H} = NDERV_{t-1}^{H} + NDERV_{tr,t}^{H} + NDERV_{rv,t}^{H} \qquad \text{(equation 83)}$$

$$NTCRED_{t}^{H} = NTCRED_{t-1}^{H} + NTCRED_{tr,t}^{H} + NTCRED_{rv,t}^{H} \qquad \text{(equation 84)}$$

The identities in the above set of equations have the same intuition as discussed earlier in the case of NFC. That is, when current transactions and revaluations are added to the past value of a financial stock, we get the present value of the financial stock.

This completes the description of the factors influencing households balance sheet in our model. To sum-up, for the household sector, we have modeled two financial stocks namely equities (assets) and loans (liabilities); both collectively constitute a substantial portion of the households balance sheet. In the next, we discuss the role of financial corporations.

5.3.3. Financial corporations

We first define the income of financial corporations. Using the definition of net income presented in section 4.2.1 and appendix 8.3 we can calculate the income received by the financial sector (Y_{t}^{FC}) using the following identity:

$$Y_t^{FC} = B2_t^{FC} + NINT_t^{FC} + NDIV_t^{FC} + NOIR_t^{FC} + NREFDI_t^{FC} + SCON_{r,t}^{FC} - SBEN_{p,t}^{FC} + OCT_t^{FC} \quad \text{(equation 85a)}$$

The income consists of various flows: gross operating surplus ($B2_t^{FC}$), net income on financial assets ($NINT_t^{FC}, NDIV_t^{FC}, NOIR_t^{FC}, NREFDI_t^{FC}$), social contributions received ($SCON_{r,t}^{FC}$), and net other current transfers (OCT_t^{FC}). Finally, we subtract social benefits paid ($SBEN_{p,t}^{FC}$) to compute net income.

By subtracting taxes from net income, we can compute disposable income as follows:

$$YD_t^{FC} = Y_t^{FC} - ITAX_t^{FC} \quad \text{(equation 85b)}$$

We can now calculate the savings of the financial corporations, which requires adjusting net disposable income for flows pertaining to pensions. The saving identity of the financial corporations can be presented as follows:

$$S_t^{FC} = YD^{FC} - PEN_t^{adj} \quad \text{(equation 86)}$$

Note that the pension deductions from the income of financial corporations are, in turn, received by the households (PEN_t^{adj}), as was shown in the savings equation for the household sector (see equation 74).

We can calculate the net lending of the financial corporations using the same set-up as before:

$$NL_t^{FC} = S_t^{FC} - I_t^{FC} - \Delta INVENT_t^{FC} - NP_t^{FC} - CT_t^{FC} \quad \text{(equation 87)}$$

Focusing on the financial aspects of the sector, we can calculate the financial net lending as follows:

$$FNL_t^{FC} = NGOLD_{t,tr}^{FC} + NDEPO_{t,tr}^{FC} + NSEC_{t,tr}^{FC} + NLOA_{t,tr}^{FC} + NEQ_{t,tr}^{FC} + NINSU_{t,tr}^{FC} + NDERV_{t,tr}^{FC} + NTCRED_{t,tr}^{FC} \quad \text{(equation 88)}$$

where FNL_t^{FC} is the financial balance which, by definition, is equal to the sum of net transactions associated with each financial stock. The only additional transaction is related to the acquisition or sale of gold denoted by $NGOLD_{t,tr}^{FC}$.

We can now define the net financial wealth (representing the net value of the financial balance sheet) of the financial sector as follows:

$$FNW_t^{FC} = NGOLD_t^{FC} + NDEPO_t^{FC} + NSEC_t^{FC} + NLOA_t^{FC} + NEQ_t^{FC} + NINSU_t^{FC} + NDERV_t^{FC} + NTCRED_t^{FC} \quad \text{(equation 89)}$$

We assume that financial corporations clear the market for deposits, loans, insurance, and derivatives. Thus, we can write:

$$NDEPO_{tr,t}^{FC} = -\left(NDEPO_{tr,t}^{NFC} + NDEPO_{tr,t}^{H} + NDEPO_{tr,t}^{G} + NDEPO_{tr,t}^{ROW}\right) \quad \text{(equation 90)}$$

$$NLOA_{tr,t}^{FC} = -\left(NLOA_{tr,t}^{NFC} + NLOA_{tr,t}^{H} + NLOA_{tr,t}^{G} + NLOA_{tr,t}^{ROW}\right) \quad \text{(equation 91)}$$

$$NINSU_{tr,t}^{FC} = -\left(NINSU_{tr,t}^{NFC} + NINSU_{tr,t}^{H} + NINSU_{tr,t}^{G} + NINSU_{tr,t}^{ROW}\right) \quad \text{(equation 92)}$$

$$NDERV_{tr,t}^{FC} = -\left(NDERV_{tr,t}^{NFC} + NDERV_{tr,t}^{H} + NDERV_{tr,t}^{G} + NDERV_{tr,t}^{ROW}\right) \quad \text{(equation 93)}$$

Using the transactions in the above set of equations, we can determine the value of each financial stock as follows:

$$NDEPO_{t}^{FC} = NDEPO_{t-1}^{FC} + NDEPO_{tr,t}^{FC} + NDEPO_{rv,t}^{FC} \quad \text{(equation 94)}$$

$$NLOA_{t}^{FC} = NLOA_{t-1}^{FC} + NLOA_{tr,t}^{FC} + NLOA_{rv,t}^{FC} \quad \text{(equation 95)}$$

$$NINSU_{t}^{FC} = NINSU_{t-1}^{FC} + NINSU_{tr,t}^{FC} + NINSU_{rv,t}^{FC} \quad \text{(equation 96)}$$

$$NDERV_{t}^{FC} = NDERV_{t-1}^{FC} + NDERV_{tr,t}^{FC} + NDERV_{rv,t}^{FC} \quad \text{(equation 97)}$$

To ensure consistency between sectoral balance (equation 87) and financial balance (equation 88), we model net transactions of securities as a residual:

$$NSEC_{tr,t}^{FC} = NL_{t}^{FC} + NL_{adj,t}^{FC} - (NGOLD_{t,tr}^{FC} + NDEPO_{tr,t}^{FC} + NLOA_{tr,t}^{FC} + NINSU_{tr,t}^{FC} + NDIR_{tr,t}^{FC} + NCRED_{tr,t}^{FC} + NEQ_{tr,t}^{FC}) \quad \text{(equation 98)}$$

Using the transactions of securities in equation 98, we can define the evolution of net stock of securities over time as follows:

$$NSEC_{t}^{FC} = NSEC_{t-1}^{FC} + NSEC_{tr,t}^{FC} + NSEC_{rv,t}^{FC} \quad \text{(equation 99)}$$

We are now left with two financial stocks on the balance sheets of financial corporations, gold reserves and the net stock of trade credits. These two financial stocks are assumed to be fully exogenous and are given by the following equations:

$$NGOLD_{t}^{FC} = NGOLD_{t-1}^{FC} + NGOLD_{tr,t}^{FC} + NGOLD_{rv,t}^{FC} \quad \text{(equation 100)}$$

$$NTCRED_{t}^{FC} = NTCRED_{t-1}^{FC} + NTCRED_{tr,t}^{FC} + NTCRED_{rv,t}^{FC} \quad \text{(equation 101)}$$

This concludes the factors in the balance sheet of the financial corporations. To sum-up, the main role of this sector is to clear the market for deposits, loans, insurance, and derivatives in the model. In the following, we describe the income and financial aspects of the government sector.

5.3.4. Government

The main income received by the government sector is taxes, this includes commodity taxes, value added taxes, other production taxes, and import duties paid by each industry. To calculate the total taxes paid to the government, we take the sum of different taxes across industries and the final demand components.

$$CT_t^{tot} = \sum_{n=1}^{9} CT_t^n + C_{ctax,t}^{tot} + GOV_{ctax,t} + INV_{ctax,t}^{tot} + INVENT_{ctax,t}^{tot} + X_{ctax,t}^{tot} \quad \text{(equation 102a)}$$

$$VAT_t^{tot} = \sum_{n=1}^{9} VAT_t^n + C_{VAT,t}^{tot} + GOV_{VAT,t}^{tot} + INV_{VAT,t}^{tot} + INVENT_{VAT,t}^{tot} + X_{VAT,t}^{tot} \quad \text{(equation 102b)}$$

$$OPT_t^{tot} = \sum_{n=1}^{9} OTP_{tax,t}^n \quad \text{(equation 102c)}$$

$$M_{duty,t}^{tot} = \sum_{n=1}^{9} M_{duty,t}^n + M_{duty,t}^C + M_{duty,t}^{GOV} + M_{duty,t}^{INV} + M_{duty,t}^X \quad \text{(equation 102d)}$$

As a result, we have total commodity taxes (CT_t^{tot}), total value added taxes (VAT_t^{tot}), total other production taxes (OPT_t^{tot}), and total import duties ($M_{duty,t}^{tot}$) received by the government.

We can add commodity taxes, value added taxes, and import taxes to calculate the net production taxes received by the government sector ($NTAX_t^{prod}$).

$$NTAX_t^{prod} = CT_t^{tot} + VAT_t^{tot} + M_{duty,t}^{tot} \quad \text{(equation 103a)}$$

As the import duties received by the government should be re-distributed to the rest of the world, these are included in the import taxes paid ($IMTAX_{p,t}$). The import taxes consist of both the total import duties ($M_{duty,t}^{tot}$) but also other import taxes paid ($OIMTAX_{p,t}$), the latter being exogenous.

$$IMTAX_{p,t} = M_{duty,t}^{tot} + OIMTAX_p \quad \text{(equation 103b)}$$

Finally, we can calculate the net import taxes received by Denmark ($NIMTax_t$) by subtracting import taxes paid ($IMTAX_{p,t}$) from the imported taxes received ($IMTAX_r$) which is also exogenous.

$$NIMTax_t = IMTAX_r - IMTAX_p \quad \text{(equation 103c)}$$

Thereby, changes in the net income taxes received by the Danish government sector ($NIMTax_t$) is only affected by changes in the import duties ($M^{tot}_{duty,t}$).

Apart from production taxes, the government also receives income taxes. We calculate the income taxes using a tax rate obtained via simple OLS regression:

$$ITAX^H_t = 0.36 * Y^H_t \quad \text{(equation 104a)}$$

$$ITAX^{NFC}_t = 0.12 * Y^{NFC}_t \quad \text{(equation 104b)}$$

$$ITAX^{FC}_t = 0.08 * Y^{FC}_t \quad \text{(equation 104c)}$$

We can then model the income taxes received by the government sector as follows:

$$ITAX^{tot}_t = ITAX^H_t + ITAX^{NFC}_t + ITAX^{FC}_t + NITAX^{ROW}_t \quad \text{(equation 105)}$$

We can now define the disposable income for the government sector as follows:

$$YD^{GOV}_t = B2^{GOV}_t + NTAX^{prod}_t + OPT^{prod}_t + NIMTax_t + ITAX^{GOV,r}_t + NINT^{GOV}_t + NDIV^{GOV}_t + NOIR^{GOV}_t + NREFDI^{GOV}_t + SCON^{GOV}_{r,t} - SBEN^{GOV}_{p,t} + OCT^{GOV}_t \quad \text{(equation 106)}$$

The government sector income consists of the following flows: gross operating surplus ($B2^{GOV}_t$)[45], net tax income ($NTAX^{prod}_t + OPT^{prod}_t + NIMTax_t + ITAX^{GOV}_t$), net income on financial assets ($NINT^{GOV}_t + NDIV^{GOV}_t + NOIR^{GOV}_t + NREFDI^{GOV}_t$), social contributions received ($SCON^G_{r,t}$), and social benefits paid ($SCON^G_{r,t}$), and finally, net other current transfers received (OCT^{FC}_t).

We define public savings as the difference between disposable income and the government consumption:

$$S^{GOV}_t = YD^{GOV}_t - G^{agg}_t \quad \text{(equation 107)}$$

We calculate the net lending for the government sector using the same method as for other sectors:

$$NL^G_t = S^{GOV}_t - I^{GOV}_t - \Delta INVENT^{GOV}_t - NP^{GOV}_t - CT^{GOV}_t \quad \text{(equation 108)}$$

We now reach the financial side, where we include the financial transactions to model financial net lending (FNL^{GOV}_t).

[45] In the national accounts, gross operating surplus is equal to the depreciation of the capital stock for the government.

$$FNL_t^{GOV} = NDEPO_{t,tr}^{GOV} + NSEC_{t,tr}^{GOV} + NLOA_{t,tr}^{GOV} + NEQ_{t,tr}^{GOV} + NINSU_{t,tr}^{GOV} + NDERV_{t,tr}^{GOV} + NTCRED_{t,tr}^{GOV} \qquad \text{(equation 109)}$$

We can now define the net financial wealth (representing the net value of the financial balance sheet) of the government sector:

$$FNW_t^{GOV} = NDEPO_t^{GOV} + NSEC_t^{GOV} + NLOA_t^{GOV} + NEQ_t^{GOV} + NINSU_t^{GOV} + NDERV_t^{GOV} + NTCRED_t^{GOV} \qquad \text{(equation 110)}$$

To ensure consistency between financial net lending and net lending, we model net transactions of securities as a residual in the model:

$$NSEC_{tr,t}^{G} = NL_t^G + NL_{adj,t}^G - (NGOLD_{t,tr}^G + NDEPO_{tr,t}^G + NLOA_{tr,t}^G + NINSU_{tr,t}^G + NDIR_{tr,t}^G + NCRED_{tr,t}^G + NEQ_{tr,t}^G) \qquad \text{(equation 111a)}$$

The net stock of securities for the government sector is defined as follows:

$$NSEC_t^G = NSEC_{t-1}^G + NSEC_{tr,t}^G + NSEC_{rv,t}^G \qquad \text{(equation 111b)}$$

The remaining financial assets for the government sector are assumed to be exogenous:

$$NDEPO_t^G = NDEPO_{t-1}^G + NDEPO_{tr,t}^G + NDEPO_{rv,t}^G \qquad \text{(equation 112)}$$

$$NLOA_t^G = NLOA_{t-1}^G + NLOA_{tr,t}^G + NLOA_{rv,t}^G \qquad \text{(equation 113)}$$

$$NEQ_t^G = NEQ_{t-1}^G + NEQ_{tr,t}^G + NEQ_{rv,t}^G \qquad \text{(equation 114)}$$

$$NINSU_t^G = NINSU_{t-1}^G + NINSU_{tr,t}^G + NINSU_{rv,t}^G \qquad \text{(equation 115)}$$

$$NDERV_t^G = NDERV_{t-1}^G + NDERV_{tr,t}^G + NDERV_{rv,t}^G \qquad \text{(equation 116)}$$

$$NGOLD_t^G = NGOLD_{t-1}^G + NGOLD_{tr,t}^G + NGOLD_{rv,t}^G \qquad \text{(equation 117)}$$

$$NCRED_t^G = NCRED_{t-1}^G + NCRED_{tr,t}^G + NCRED_{rv,t}^G \qquad \text{(equation 118)}$$

A government surplus (deficit) will lower (increase) the number of securities issued by the government sector. We now proceed to explaining the rest of the world, which is the last institutional sector of the model.

5.3.5. Rest of the world

The main income flows received by the rest of the world (YD_t^{ROW}) consists of the net imports of Denmark ($M_t^{tot} - X_t^{tot}$), net wages (W_t^{ROW}), income and import taxes ($NIMTax_t + NITAX_t^{ROW}$), net financial income ($NINT_t^{ROW} + NDIV_t^{ROW} + NOIR_t^{ROW} + NREFDI_t^{ROW}$), net social contributions and benefits ($NSCON_{r,t}^{ROW} - NSBEN_{p,t}^{ROW}$), and finally net other current transfers (OCT_t^{ROW}). The disposable income from the perspective of the rest of the world can be defined as follows:

$$\begin{aligned} YD_t^{ROW} = {} & M_t^{tot} - X_t^{tot} + W_t^{ROW} - (NIMTax_t + NITAX_t^{ROW}) + NINT_t^{ROW} + NDIV_t^{ROW} \\ & + NOIR_t^{ROW} + NREFDI_t^{ROW} + NSCON_{r,t}^{ROW} - NSBEN_{p,t}^{ROW} \\ & + OCT_t^{ROW} \end{aligned} \qquad \text{(equation 119)}$$

Net lending for the rest of the world (NL_t^{ROW}), follows the same set-up used for other sectors.

$$NL_t^{ROW} = YD_t^{ROW} - CT_t^{ROW} - NP_t^{ROW} \qquad \text{(equation 120)}$$

For the financial side of the sector, we can start by calculating financial net lending (FNL_t^{ROW}), accounting for the net financial transactions:

$$\begin{aligned} FNL_t^{ROW} = {} & NGOLD_{t,tr}^{ROW} + NDEPO_{t,tr}^{ROW} + NSEC_{t,tr}^{ROW} + NLOA_{t,tr}^{ROW} + NEQ_{t,tr}^{ROW} + NINSU_{t,tr}^{ROW} \\ & + NDERV_{t,tr}^{ROW} + NTCRED_{t,tr}^{ROW} \end{aligned} \qquad \text{(equation 121)}$$

Next, we define net financial wealth of the rest of the world, determined by the net stocks of assets held by the rest of the world:

$$\begin{aligned} FNW_t^{ROW} = {} & NGOLD_t^{ROW} + NDEPO_t^{ROW} + NSEC_t^{ROW} + NLOA_t^{ROW} + NEQ_t^{ROW} + NINSU_t^{ROW} \\ & + NDERV_t^{ROW} + NTCRED_t^{ROW} \end{aligned} \qquad \text{(equation 122)}$$

In the model structure, consistency for securities and trade credits is achieved by assigning a passive role to transactions pertaining to the rest of the world as follows:[46]

$$NSEC_{tr,t}^{ROW} = -\left(NSEC_{tr,t}^{NFC} + NSEC_{tr,t}^{H} + NSEC_{tr,t}^{G} + NSEC_{tr,t}^{FC}\right) \qquad \text{(equation 123a)}$$

$$NSEC_t^{ROW} = NSEC_{t-1}^{ROW} + NSEC_{tr,t}^{ROW} + NSEC_{rv,t}^{ROW} \qquad \text{(equation 123b)}$$

$$\begin{aligned} NTCRED_{tr,t}^{ROW} = {} & -\left(NTCRED_{tr,t}^{NFC} + NTCRED_{tr,t}^{H} + NTCRED_{tr,t}^{G} \right. \\ & \left. + NTCRED_{tr,t}^{FC}\right) \end{aligned} \qquad \text{(equation 123c)}$$

[46] As we have already defined all other net transactions of all financial assets, securities issued by the rest of the world closes the budget constraint of financial corporations. To not overidentify the model, we exclude the equation ensuring consistency in the security market.

$$NTCRED_t^{ROW} = NTCRED_{t-1}^{ROW} + NTCRED_{tr,t}^{ROW} + NTCRED_{rv,t}^{ROW} \quad \text{(equation 123d)}$$

To fulfil vertical consistency for the rest of the world, we model the transactions in net deposits as follows:

$$NDEPO_{tr,t}^{ROW} = NL_t^{ROW} + NL_{adj,t}^{ROW} - (NGOLD_{t,tr}^{ROW} + NDEPO_{tr,t}^{ROW} + NLOA_{tr,t}^{ROW} + NINSU_{tr,t}^{ROW} + NDIR_{tr,t}^{ROW} + NCRED_{tr,t}^{ROW} + NEQ_{tr,t}^{ROW}) \quad \text{(equation 123e)}$$

$$NDEPO_t^{ROW} = NDEPO_{t-1}^{ROW} + NDEPO_{tr,t}^{ROW} + NDEPO_{rv,t}^{ROW} \quad \text{(equation 123f)}$$

Finally, the rest of the financial assets are modelled as exogenous for the rest of the world as follows:

$$NDEPO_t^{ROW} = NDEPO_{t-1}^{ROW} + NDEPO_{tr,t}^{ROW} + NDEPO_{rv,t}^{ROW} \quad \text{(equation 124)}$$

$$NLOA_t^{ROW} = NLOA_{t-1}^{ROW} + NLOA_{tr,t}^{ROW} + NLOA_{rv,t}^{ROW} \quad \text{(equation 125)}$$

$$NEQ_t^{ROW} = NEQ_{t-1}^{ROW} + NEQ_{tr,t}^{ROW} + NEQ_{rv,t}^{ROW} \quad \text{(equation 126)}$$

$$NINSU_t^{ROW} = NINSU_{t-1}^{ROW} + NINSU_{tr,t}^{ROW} + NINSU_{rv,t}^{ROW} \quad \text{(equation 127)}$$

$$NDERV_t^{ROW} = NDERV_{t-1}^{ROW} + NDERV_{tr,t}^{ROW} + NDERV_{rv,t}^{ROW} \quad \text{(equation 128)}$$

$$NGOLD_t^{ROW} = NGOLD_{t-1}^{ROW} + NGOLD_{tr,t}^{ROW} + NGOLD_{rv,t}^{ROW} \quad \text{(equation 129)}$$

We now move to the third and final block of our framework, related to the environmental aspects of the economy.

5.4. Environmental aspects

In the environmental block of the model, we concentrate on three crucial aspects related to climate targets: energy (use and supply), emissions, and emission taxes. Our aim is to establish a relationship between economic activity, energy, and emissions. We focus on identifying the intensities at which various economic activities contribute to climate damage, rather than on the specific scientific mechanism through which this effect occurs. It is important to highlight that our model does not incorporate any damage function, i.e., there is no feedback mechanism through which climate change per se can impose economic costs.

5.4.1. Energy usage and supply

In our analysis, we use 21 different types of energies (see Table 5), following the same classification used in the databank for the GreenREFORM model (Svarer et al. 2024). To establish a link between energy (usage-supply) and economic activity, we assume a linear relationship. [47] Focusing on domestic energy production, we can express the total supply of domestic energy as a fraction of real production. For each industry n, this is given by the following equation (without time scripts):

$$ENERGY_{sup,t}^{n} = D_{sup,t}^{ENERGY,n} * prod_{dom,t}^{n} \qquad \text{(equation 130a)}$$

$ENERGY_{sup,t}^{n}$ represent the different types of energy supplied (in physical units) by an industry n, as a result of production; $D_{sup,t}^{ENERGY,n}$ represents the amount of energy supplied per unit of output produced by the industry. This can be understood as the industry's contribution to the energy supply relative to its production. A higher value in this case signals that the industry is supplying a larger amount of energy per unit of its output (which is the case for energy producing industry). Note that $D_{sup,t}^{ENERGY,n}$ is exogenously determined.[48] As can be observed in Table 5, the Energy production and refinery industry has the highest contribution in energy production.

To calculate total energy supply in the economy, we need to account for supply of waste $\left(ENERGY_{sup,t}^{Waste}\right)$, renewables used as input in production $\left(ENERGY_{sup,t}^{RE}\right)$, and imports of different energy types ($ENERGY_{sup,t}^{M}$) as shown in Table 5.[49]

$$ENERGY_{sup,t}^{tot} = \sum_{n=1}^{9} ENERGY_{sup,t}^{n} + ENERGY_{sup,t}^{M} + ENERGY_{sup,t}^{Waste} + ENERGY_{sup,t}^{RE} \qquad \text{(equation 130b)}$$

Focusing on energy usage, we can express the total energy usage of each industry as a fraction of its total production. For each industry n, this is given by the following equation:

$$ENERGY_{use,t}^{n} = D_{use,t}^{ENERGY,n} * prod_{t}^{n} \qquad \text{(equation 131a)}$$

[47] The simple linear relationship between output in each industry and energy usage is highly simplifying. In future versions of the model a more detailed modelling of the energy-market should be made, allowing for substitution between energy products used in production.

[48] Both the coefficients for energy supply (equation 130a) and usage (equation 131a) are calculated outside the model as: $D_{use,t}^{ENERGY,n} = \frac{ENERGY_{use,t}^{n}}{sale_{t}^{n}}$ and $D_{sup,t}^{ENERGY,n} = \frac{ENERGY_{sup,t}^{n}}{sale_{t}^{n}}$.

[49] Renewables used as inputs in production mostly covers the use of wind to produce electricity.

Where $ENERGY^{n}_{use,t}$ represents different energy types used (in physical units as expressed in Gj) by an industry in production ($sale^{n}_{t}$) which is expressed in monetary values (DKK in this case); $D^{ENERGY,n}_{use,t}$ can be seen as an indication of how efficiently energy is being used in production. A lower value can both indicate a high energy efficiency or that a specific energy type is not used in this industry's energy-mix. Take the example of oil products (*OilP*) which is used by all the nine industries. In 2019, the industry with the lowest usage of oil products per unit of output were the financial corporation industry. This is due to the energy-mix used by financial corporations and does not necessarily mean that this is the most efficient industry.

Equation 131a only covers energy used in the process of production by domestic industries. In order to compute the total energy usage of the economy, we need to take into account the energy used by households ($ENERGY^{HH}_{use,t}$), distribution losses ($ENERGY^{DL}_{use,t}$), and energy exported ($ENERGY^{X}_{use,t}$). The total energy usage of the economic can be calculated as follows:

$$ENERGY^{tot}_{use,t} = \sum_{n=1}^{9} ENERGY^{n}_{use,t} + ENERGY^{HH}_{use,t} + ENERGY^{DL}_{use,t} + ENERGY^{X}_{use,t} \qquad \text{(equation 131b)}$$

We endogenize the energy used by households using a linear relationship between energy used and domestic consumption. This relationship is expressed in equation 132:

$$ENERGY^{HH}_{use,t} = D^{ENERGY,HH}_{use,t} * c^{dom}_{t} \qquad \text{(equation 132)}$$

We use the difference between total energy usage and total energy supply, to calculate the change in energy inventories as follows:

$$Inv^{ENERGY}_{Delta,t} = ENERGY^{tot}_{sup,t} - ENERGY^{tot}_{use,t} \qquad \text{(equation 133)}$$

Unfortunately, the stock values of these inventories are not available for our sample, therefore we set the starting stock value to zero in the first period of the sample with changes in this stock being defined as:

$$Inv^{ENERGY}_{t} = Inv^{ENERGY}_{t-1} + Inv^{ENERGY}_{Delta,t} \qquad \text{(equation 134)}$$

We now focus on energy reserves available in the economy. There are two energy reserves in Denmark: Crude oil ($Coil^{gj}_{res,t}$) and Natural gas extracted ($NTgas^{gj}_{res,t}$). The data on these reserves is measured in m3 and Nm3, but we convert them to gigajoules to fit the energy usage and supply accounts. For conversions, we use the conversion rates presented in the annual reports by the

Danish energy agency. It can be shown that the reserves for crude oil and natural gas evolve according to the following equations:[50]

$$Coil^{gj}_{res,t} = Coil^{gj}_{res,t-1} - \sum_{n=1}^{9} Coil^{n}_{sup,t-1} + Coil^{gj}_{otc,t-1} \qquad \text{(equation 135a)}$$

$$NTgas^{gj}_{res,t} = NTgas^{gj}_{res,t-1} - \sum_{n=1}^{9} NTgas^{n}_{extr,sup,t-1} + NTgas^{gj}_{otc,t-1} \qquad \text{(equation 135b)}$$

In the above set of equations, $Coil^{gj}_{res,t}$ and $NTgas^{gj}_{res,t}$ are the opening stocks of crude oil and natural gas. These energy reserves will deplete over time depending on the pace of extractions, which in turn, depends on the demand for these type of energies; the extraction of crude oil is denoted by $\sum_{n=1}^{9} Coil^{sup}_{n,t-1}$ and the extraction of natural gas is denoted by $\sum_{n=1}^{9} NTgas^{sup}_{extr,n,t-1}$. New discoveries of reserves or revaluations are added to the existing stocks, captured by the terms $Coil^{gj}_{otc,t-1}$ for crude oil, and $NTgas^{gj}_{otc,t-1}$ for natural gas.[51]

5.4.2. Emissions

We now present a detailed assessment of GHG emissions in the economy. In our analysis, we begin by categorizing emissions based on economic activity. In this regard, we distinguish emissions generated in the **production process** from those generated by **household consumption**. Afterwards, both sources of emissions (i.e., production and consumption-related emissions) are further classified into two types: **i) Direct Energy-Related Emissions**, which are emissions caused by the use of energy sources, such as emissions resulting from burning fossil fuels like coal, oil, or natural gas; and **ii) Indirect Emissions**, which are emissions associated with economic activities (production and consumption of goods) but are not directly caused by energy usage, e.g., emissions from waste management practices or methane emissions by livestock.

Note that emissions originating from the aforementioned activities can take the form of specific gases. As mentioned in section 4.3.2, we include the following types of emissions: carbon dioxide

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[50] Supply data for crude oil and natural gas matches closely with the production data for crude oil and natural gas reserves provided by the Danish energy agency for oil and gas reserves in Denmark. Therefore, we assume that domestic supply data from Statistics Denmark shows how much oil and gas is produced/extracted from the oil and gas reserves. Thereby the rest of the data is consistent with consumption and use data also used from Statistics Denmark.

[51] As data on crude oil and natural gas reserves were inconsistent in 2006, 2014 and 2015, probably as a result of rounding errors, we have adjusted the data for $Coil^{gj}_{otc,t-1}$ by + 1 m3 in 2006, +1 m3 in 2014, and by -1 m3 in 2015, thereby making the data consistent. For natural gas ($NTgas^{gj}_{otc,t-1}$.), we +1 Nm3 in 1997, - 1 Nm3 in 2006, + 1 Nm3 in 2012.

(CO2), nitrous oxide (N2O), methane (CH4), sulfur hexafluoride (SF6), perfluorocarbons (PFC), and hydrofluorocarbons (HFC).[52] These forms of emissions are later used to calculate the CO2-equivalent measure.[53]

In what follows, it is helpful to understand the distinction between the sources of emissions (generated by 4 types of economic activity) and the forms they take (6 forms of GHG emissions as discussed above). In the following sections, we will first examine the direct and indirect emissions produced during the process of production. Afterward, we will address the direct and indirect emissions resulting from households' consumption.

A. Direct Energy-Related Emissions of domestic production

Following the approach of Beck and Dahl (2020), we calculate emission coefficients linked to each energy type for each industry. To that end, we use a dataset linking emissions to the use of specific energy types for each industry. The emission coefficient in the case of direct emissions for each industry is calculated as follows:

$$EMISSION_{coef,t}^{ENERGY,n} = \frac{EMISSION_{ENERGY,t}^{n}}{ENERGY_{use,t}^{n}} \qquad \text{(equation 136a)}$$

where n determines industry, $EMISSION$ determines the type of emissions (e.g. CO2, N20, and so on), and $ENERGY$ determines energy type (e.g. Crude oil, Oil products, and so on). To give an example, the energy coefficient relating the usage of oil products (OilP) to carbon dioxide emissions (CO2) in the agriculture industry is calculated as follows:

$$CO2_{coef,t}^{OilP,AGRI} = \frac{CO2_{OilP,t}^{AGRI}}{OilP_{use,t}^{AGRI}} \qquad \text{(equation 136b)}$$

where $EMISSION = CO2$, $ENERGY = OilP$, and $n = AGRI$.

Since we have emissions related to each of the 21 energy types, we can take the sum across the energy types for each industry to calculate the total direct energy-related emissions for each individual industry.

[52] SF6, PFC, and HFC are not emitted as a result of using energy, whereas they only occur for the category "unrelated to energy".
[53] The model can be extended to include additional emission types, Data from Denmark statistics allow us to include 15 types of emissions.

$$EMISSION_{DIRECT,t}^{n} = \sum_{ENERGY=1}^{21} ENERGY_{use,t}^{n} * EMISSION_{coef,t}^{ENERGY,n} \quad \text{(equation 136c)}$$

Note that the available data used to estimate emission coefficient is available until 2017. Since, our sample extends to 2019, we assume the same emission coefficients from 2017-2019.

B. Indirect emissions of domestic production

To estimate indirect emissions related to production, we calculate emission coefficients using total production of a given industry.[54] The calculation of emission coefficients in this case is given by:

$$EMISSION_{coef,t}^{INDIRECT,n} = \frac{EMISSION_{INDIRECT,t}^{n}}{prod_{t}^{n}} \quad \text{(equation 137a)}$$

Again, using n as the notation for industries, and $EMISSION$ as the notation for emission type.

Using our calculated emission coefficient in equation 137a, the total indirect emission for each industry is given by:

$$EMISSION_{INDIRECT,t}^{n} = EMISSION_{coef,t}^{INDIRECT,n} * prod_{t}^{n} \quad \text{(equation 137b)}$$

By aggregating equation 136c and 137b, we can calculate the total emission (incl. direct and indirect) for each industry as follows:

$$EMISSION_{tot,t}^{n} = EMISSION_{INDIRECT,t}^{n} + EMISSION_{DIRECT,t}^{n} \quad \text{(equation 138)}$$

C. Direct Energy-Related Emissions of households consumption

To calculate direct energy-related emissions for the household sector, we follow the same procedure as was followed for industries. In this case, the emission coefficients linked to the use of each energy type is given by:

$$EMISSION_{coef,t}^{ENERGY,HH} = \frac{EMISSION_{ENERGY,t}^{HH}}{ENERGY_{use,t}^{HH}} \quad \text{(equation 139a)}$$

The emission coefficients in equation 139a are used to calculate direct energy-related emissions for the household sector following the same approach as for industries in equation 136a. Again, we can take the sum across the energy types to calculate the total direct energy-related emission for the household sector as follows:

[54] This is similar to the methods used in the GreenREFORM model by the DREAM group.

$$EMISSION_{DIRECT,t}^{HH} = \sum_{ENERGY=1}^{21} ENERGY_{use,t}^{HH} * EMISSION_{coef,t}^{ENERGY,HH} \quad \text{(equation 139b)}$$

D. Indirect emissions of households consumption

To estimate indirect emission for the household sector, we calculate emission coefficients using domestic consumption. The emission coefficients in this case are given by:

$$EMISSION_{coef,t}^{INDIRECT,HH} = \frac{EMISSION_{INDIRECT,t}^{HH}}{c_t^{dom}} \quad \text{(equation 140a)}$$

The emissions coefficients from equation 140a are then used to estimate the indirect emissions associated with consumption:

$$EMISSION_{INDIRECT,t}^{HH} = EMISSION_{coef,t}^{INDIRECT,HH} * c_t^{dom} \quad \text{(equation 140b)}$$

By aggregating equation 139b and 140b, we can find the total emissions generated by households consumption as follows:

$$EMISSION_{tot,t}^{HH} = EMISSION_{DIRECT,t}^{HH} + EMISSION_{INDIRECT,t}^{HH} \quad \text{(equation 141)}$$

We can now find the total emission for each emission type in the entire economy by aggregating equation 138 and 141. That is, we add the total emissions from the 9 industries together with emissions from households:

$$EMISSION_t^{tot} = \sum_{n=1}^{9} EMISSION_{tot,t}^{n} + EMISSION_{tot,t}^{HH} \quad \text{(equation 142)}$$

As we now have total emissions for each of the 6 emission types, we can now convert each form of emission into CO2 equivalent using the commonly used GWP conversion rates. More specifically, for CO2, SF6, PFC, and HFC the conversion rate is 1 as they are already measured in CO2-equivelants; for CH4 the conversion rate is 25, and for N2O the conversion rate is 298. We calculate the CO2-equivelant emissions (CO2E) first at an industry level:

$$CO2E_t^n = CO2_t^n + SF6_t^n + PFC_t^n + HFC_t^n + 25 * CH4_t^n + 298 * N2O_t^n \quad \text{(equation 143a)}$$

And for the entire economy:

$$CO2E_t^{tot} = CO2_t^{tot} + SF6_t^{tot} + PFC_t^{tot} + HFC_t^{tot} + 25 * CH4_t^{tot} + 298 * N2O_t^{tot} \quad \text{(equation 143b)}$$

This completes the description of the environmental block of the model, where we have modelled the nexus between energy, emissions and economic activity. We now proceed to discussing an important policy variable, environmental taxes, that is recently receiving a lot of attention in the discussions related to climate targets.

5.4.3. Environmental taxes

In section 5.1.2, we isolated environmental taxes from the rest of other production taxes (see equation 12). In equation 144a we assume environmental taxes to be dependent on CO2E emissions of the industry as follows:[55]

$$ENV_{tax,t}^{n} = CO2E_{rate,t}^{n} * CO2E_{t}^{n} \qquad \text{(equation 144a)}$$

We use an exogenously calculated tax rate for each industry n ($CO2_{rate,t}^{n}$), which is calculated outside the model using the following equation:

$$CO2E_{rate,t}^{n} = \frac{ENV_{tax,pollution}^{n}}{CO2E_{t}^{n}} \qquad \text{(equation 144b)}$$

This completes the environmental aspects of the model and also the entire model description. We are now ready to evaluate the model, after which we will perform three simple scenarios.

6. Model evaluation

To evaluate the model, we perform two standard checks. First, we numerically solve the model to establish a baseline, the results of which are compared with the observed data. Second, we analyse the response of the model to several shocks to analyse whether or not the model is capable of capturing the stylised facts. This step is also crucial for understanding the different transmission mechanisms embedded in the model.

Figure 2 shows the prediction of the model for real production of domestic industries in domestic currency. The model decently captures the overall tendency and fluctuation within each of the nine industries.

Figure 2: Total real production of domestic industries

[55] Equation 144a relates CO2-equivelent emissions directly to the environmental taxes paid by a given industry. In reality, most environmental taxes are paid using energy related taxes where a specific tax rate is put on energy usage. In this current version of the model, we rely on the simpler set-up of equation 144a whereas this should be modeled in more detail in future versions.

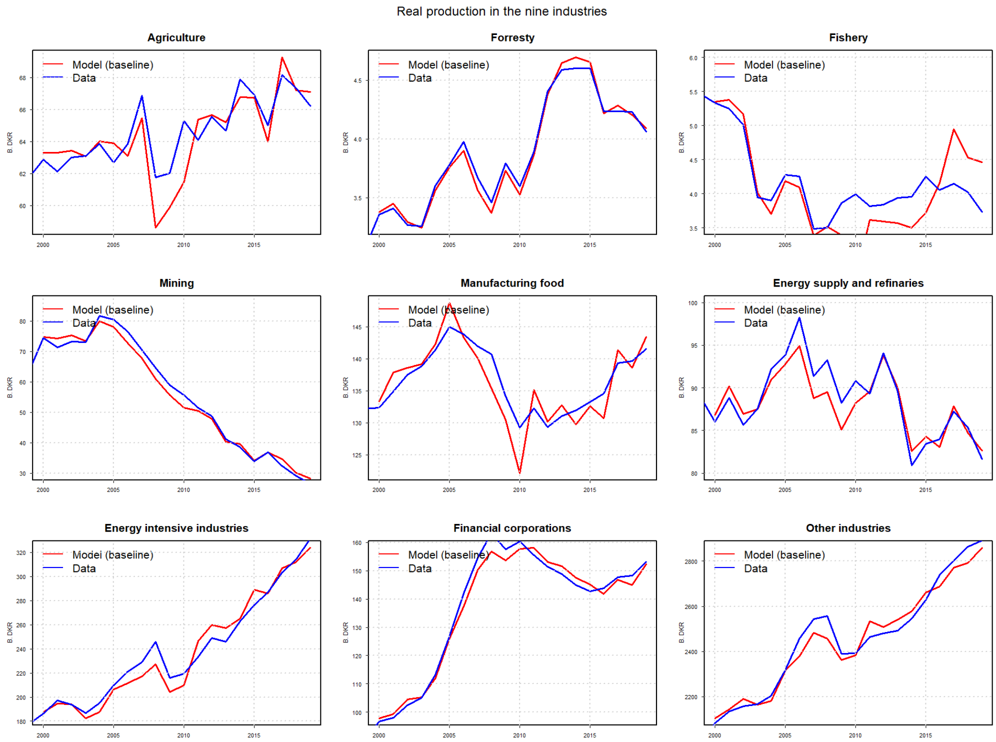
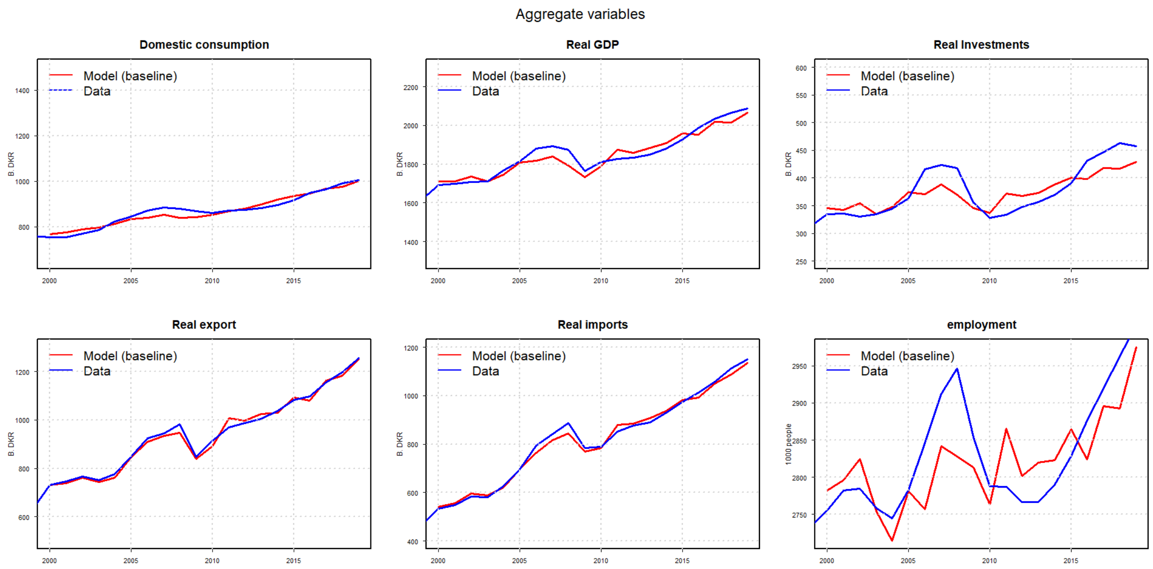


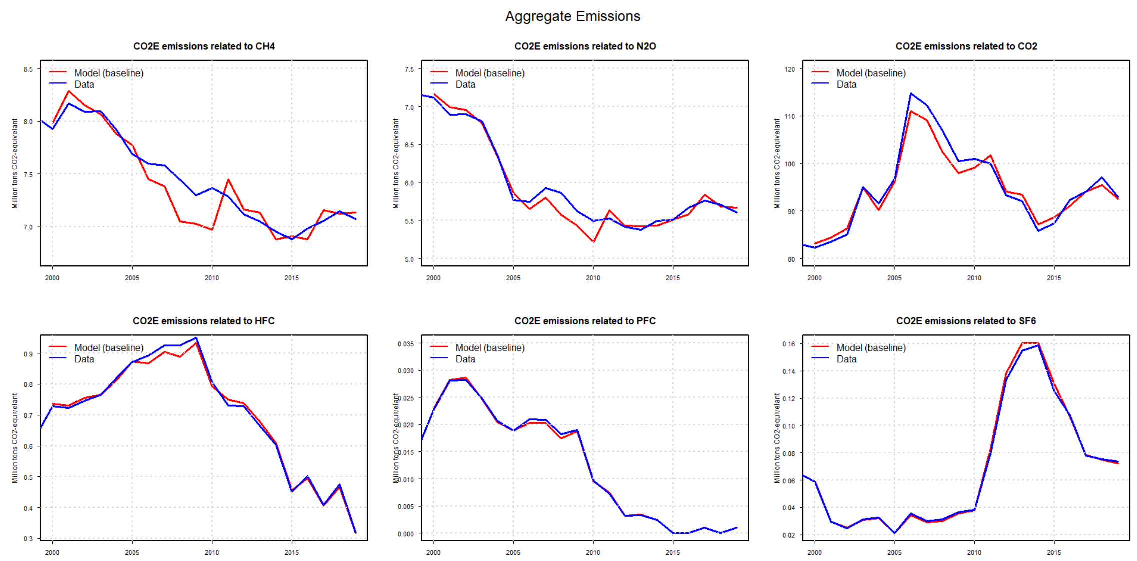
Figure 3 shows the development of real GDP (including its components) and employment. Again, we can see that the model captures the development of key macroeconomic variables. There are some episodes of divergences between the model predictions and original data for employment, but the model performs fairly well in capturing both the long run tendency as well as cyclical movements.

Figure 3: GDP components and employment



We now present the model performance related to the environmental aspects of the model. Figure 4 shows the model prediction of total CO2 equivalent emissions associated with the 6 emission types. The prediction of the model for emissions is consistent with the original data; it decently captures both the trend and fluctuations. We can conclude that the model performs reasonably well in capturing the development in key variables related to the economy and environment.

Figure 4: Emissions



To ensure that our model fulfils the properties of a stock-flow-consistent model, we need to ensure that each financial asset has a counterparty, and that each transaction has an origin and a destination. First, to ensure consistency in financial assets, we need to check that net holdings of financial stocks across the institutional sectors sum to zero, implying that the holding of every financial asset has a counterparty, or put differently, someone's financial asset is someone's financial liability. In principle, one can perform this check on each financial stock, but in practice, performing the check on the net holding of securities will suffice in our case. The reason is that transactions related to securities by the rest of the world ($NSEC_{tr,t}^{ROW}$) are not included as an equation in the model, as this is our redundant equation. All other financial stocks and their related transactions, through specific accounting identities, are tied across the sectors through explicit equations in the model. Therefore, if net holdings of securities across the sectors sum to zero, we can safely conclude that the model fulfils the requirement of stock consistency. Second, we need to ensure that the sum of net lendings (or financial balances) across sectors sum to zero, meaning that a sector's deficit is financed by other sectors' surplus. After carrying out the aforementioned-tests, we find that both conditions are fulfilled, and the model is stock-flow-consistent.

6.1. Introducing three simple scenarios

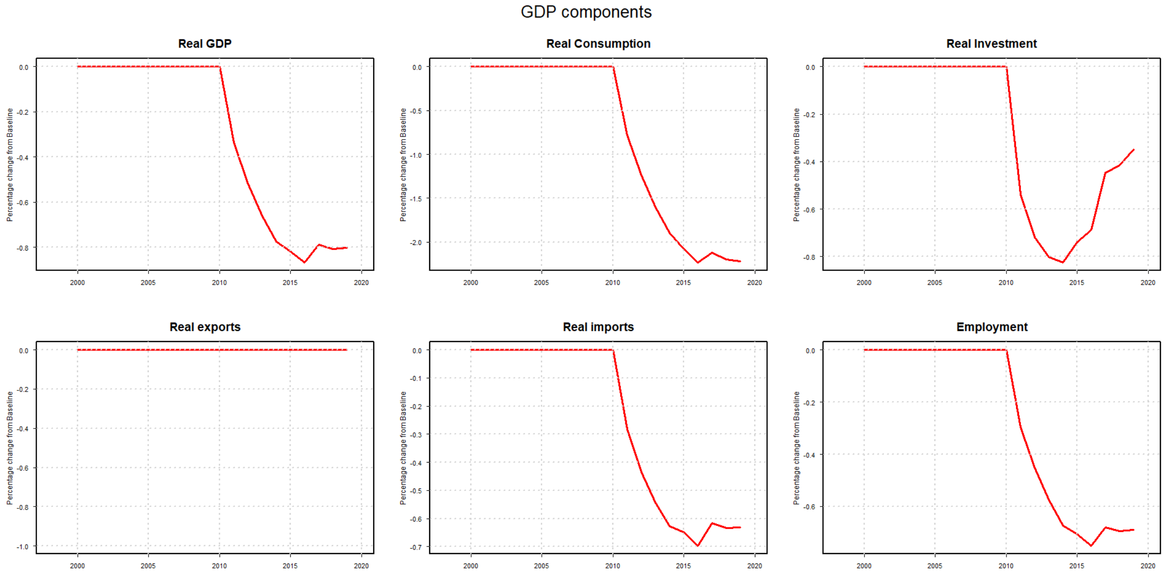
We now analyse the response of the model to various types of shocks related to monetary and fiscal policy. First, we introduce a **monetary policy shock**, in which we permanently increase the interest rates on loans, deposits and bonds by 2 percentage points. Second, we introduce 2 types of fiscal policy shocks as follows: **i) public spending shock,** where we permanently increase government consumption by 5% in the biggest industry (categorised as "other industries"), and **ii) carbon tax shock**, in which we permanently increase the carbon tax rate in the agricultural industry.

The effects of the shocks are presented as deviations from the baseline. Note that these shocks are introduced independently of each other in the baseline in 2010. For instance, when we introduce a carbon tax shock to the model, we do not introduce any other shock in the same scenario.

6.1.1. A shock to interest rates

The effects of a contractionary monetary policy shock on gross domestic product (GDP) components are visualised in Figure 5. Following a monetary policy shock, GDP contracts by about 0.8% relative to the baseline value; this development is mainly driven by a fall in consumption (which reduces by 2.1%). The main mechanism is that higher interest rates lower disposable income, which causes final consumption to fall. The overall contraction in the economy also has the effect of lowering investment and employment; trade balance slightly improves as imports contract sharply whereas exports remain unaffected.

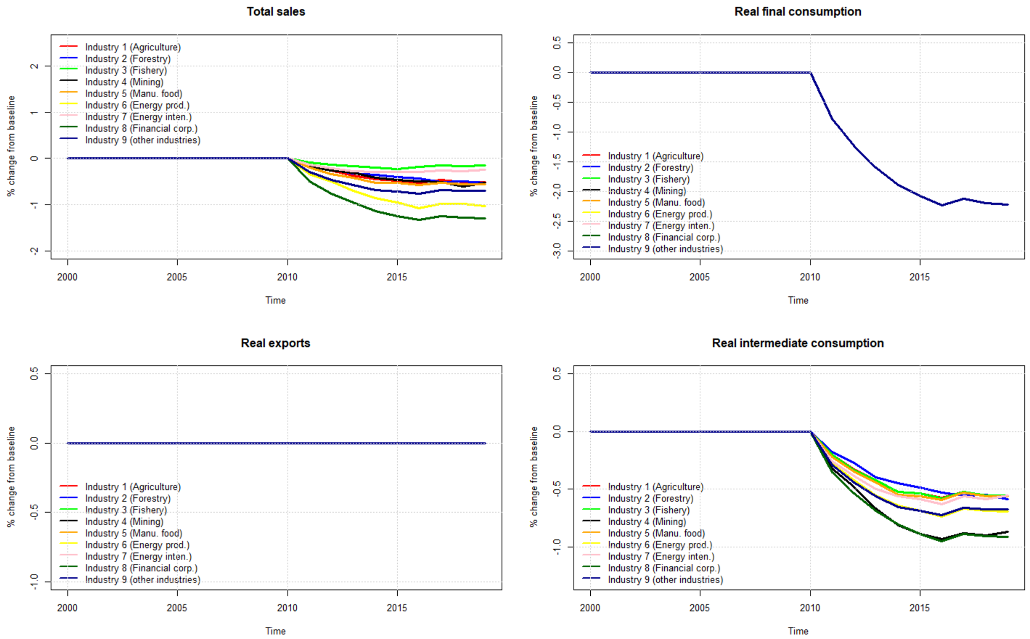
Figure 5: Change in GDP components



In Figure 6, we show the effects of the shock on real production along with some of the key components for each industry. We can observe that an increase in interest rate has the effect of lowering real production across all industries. The main transmission channel is that the fall in final consumption lowers production, which in turn, reduces intermediate consumption. The reduction in intermediate consumption reinforces the reduction in total production. It is interesting to note that the effects of the shock on total production across the industries are heterogenous (i.e., financial corporations experience a larger drop in real production compared to other industries), but the effects of the shock on final consumption are homogenous, i.e., the final consumption of the goods supplied by each industry falls with the same magnitude. The reason is that income shocks (in this case induced by higher interest rates) affect the final consumption proportionally. The heterogenous response of total production across the industries is partly driven by their sales of inputs to other industries (which in turn, depend on the production requirement of the industry) and partly by the difference in weights of the underlying components of total production.

Figure 6: Changes in real production for industries

Industry production components



We now proceed to discussing the effects of fiscal shocks in our model.

6.1.2. A shock to government spending

We introduce a government spending shock, characterized by a 5% increase in government consumption of final goods supplied by industry no. 9 called "other industries". In figure 7, we depict the effects of this shock on GDP, along with several key macroeconomic components. We can see that the shock triggers expansionary economic effects; specifically, we observe an increase in final consumption and real investments, reflecting the positive spillover effects of higher government demand on private sector activity. Moreover, we find the fiscal multiplier for this shock to be around 1.14.

Figure 7: Changes in GDP and underlying components

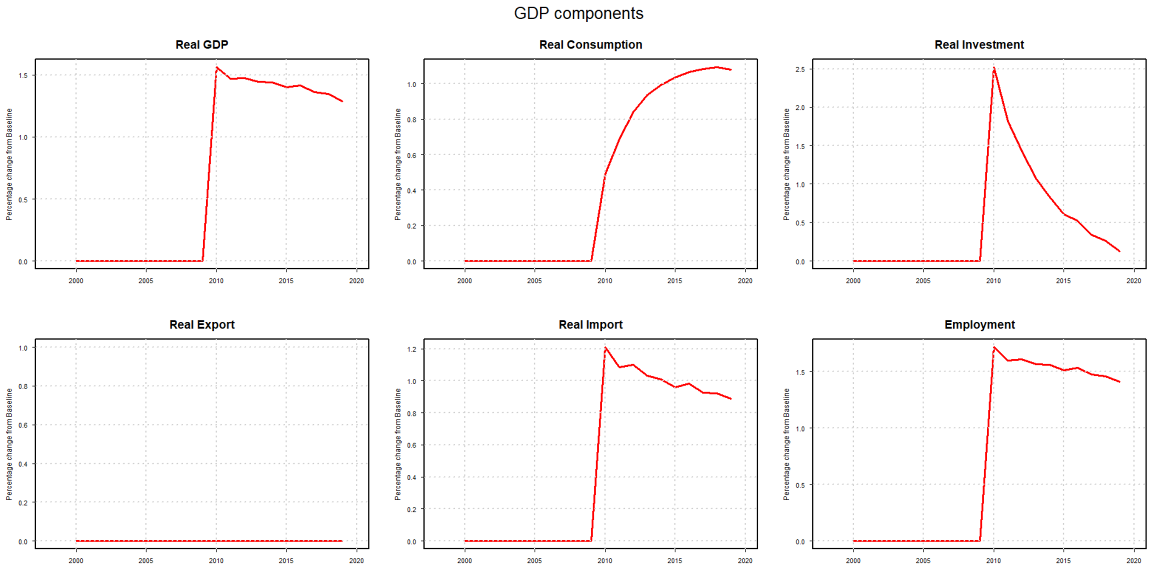
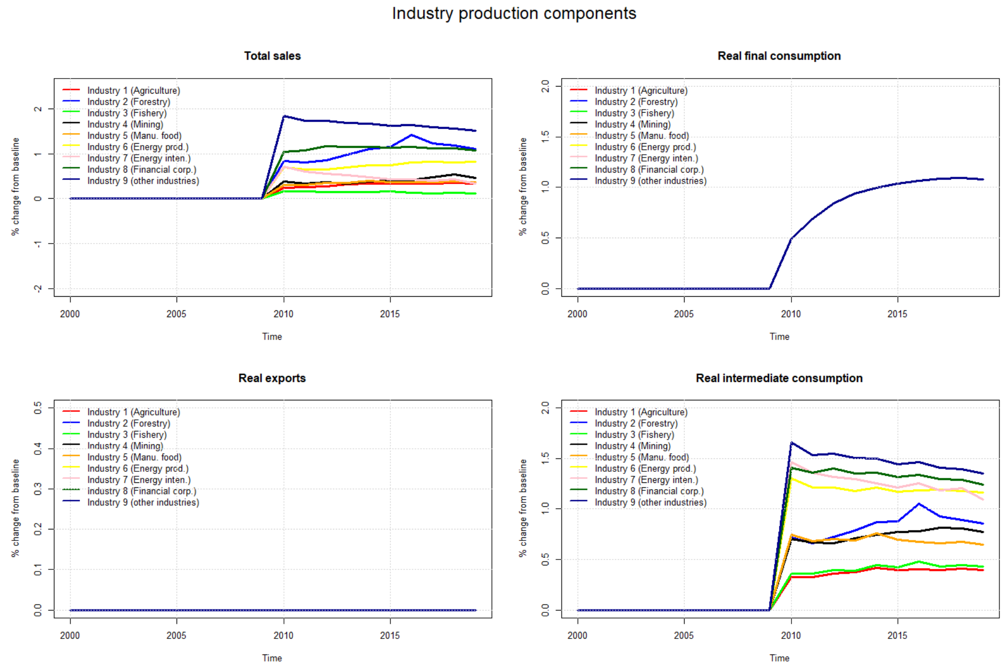


Figure 8 visualises the effects of government spending shock on the production of industries (including the main components of production). Once again, the expansionary effects of government spending can be observed as total production increase for all industries. The expansionary effects of the shock on total production can be explained by two main channels: i) The increase in government spending increases the production requirement, which in turn, induces the consumption of intermediate goods, and ii) the overall increase in income (as shown in Figure 7) increases the aggregate final consumption of the goods. These two effects combined can explain the increase in total production across the industries. While the shock has a general expansionary effect, we can once again observe that the effects (in terms of its magnitude) on total production vary across the industries. This difference is partly explained by the degree of dependency of industry 9 (other industries) for its inputs on the remaining industries (captured through the technical coefficients) and partly by the difference in weights of the underlying components of total production. Note that the real exchange rate remains unaffected, and therefore we observe no effect on exports in the industries.

Figure 8: Changes in real production and underlying components



In the next section we will introduce a contrary fiscal policy shock, as we increase carbon taxes in the agricultural industry.

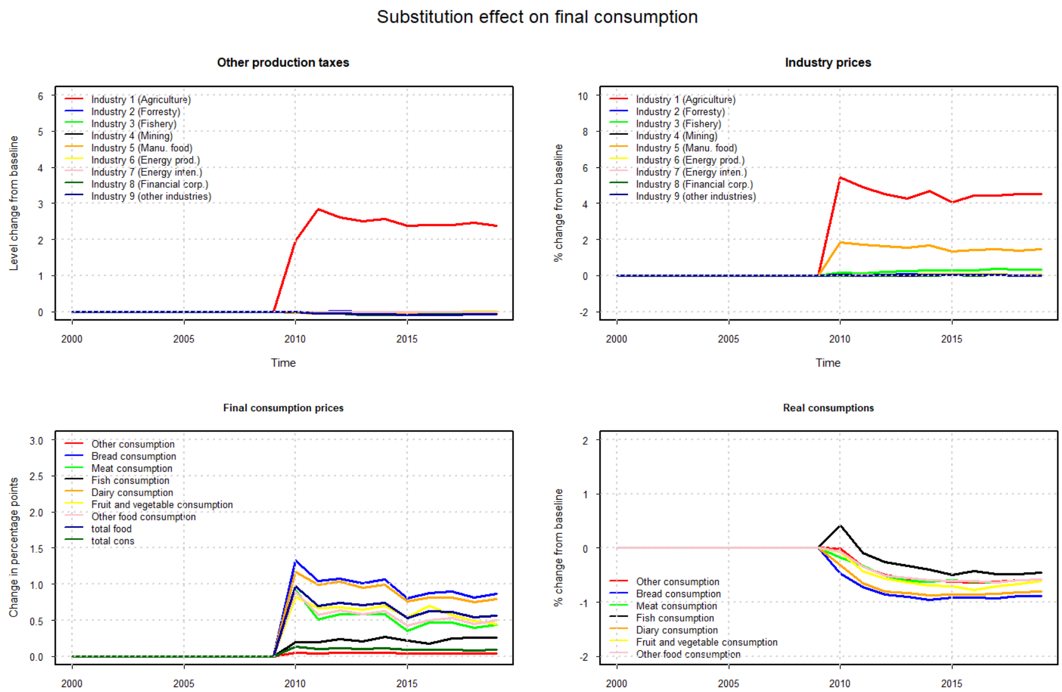
6.1.3. A shock to carbon taxes

In this section, we analyse the mechanism through which carbon taxes affect the level of economic activity and GHG emissions. In section 5.4.3, we assumed that punitive measures on carbon emissions can take the form of environmental taxes. We linked environmental taxes to CO2-equivelant emissions, using the tax rate $\mathrm{CO2E}^{n}_{rate,t}$ in equation 144a. To introduce a carbon tax shock for an industry, we can increase the tax rate $\mathrm{CO2E}^{n}_{rate,t}$ for an industry of our choice. In this paper, we choose to increase the carbon tax for the agriculture industry, as this topic has recently received a lot of attention in policy discussions. Consequently, the Danish government has recently reached an agreement with the organization of Danish agriculture to introduce a carbon tax in the agricultural sector. The proposed tax will be implemented in 2030 and is planned to reach 350 DKK per ton of CO2E emissions, excluding those related to energy use, by 2035.

Against this background, we discuss the implementation of a carbon tax in the agriculture industry. In the baseline model the agricultural sector is estimated to pay a carbon tax of approximately 120 DKK for each ton of CO2 emissions. We increase this tax rate to the level suggested by the government agreement, i.e., 350 DKK for each ton of CO2E emissions unrelated to energy. Since we introduced the shock in 2010, our results should not be interpreted as reflective of the current scenario. We merely perform this shock to explain the mechanism through which these type of policy measures can affect the economy and GHG emissions.

Figure 9 illustrates the impact of a carbon tax shock on tax expenditure, industry and consumer prices, as well as the substitution effects in final consumption. As the carbon tax increases production costs in the agricultural sector, these additional costs are partially passed onto other industries through higher input prices, which ultimately raises final consumer prices. Dairy and bread products experience the most significant price increases, as their production is linked to the agriculture and food manufacturing sectors, both of which experience substantial cost hikes. In response to price increases, consumers tend to substitute these products with other food items.

Figure 9: Price effects and substitution between product types



In Figure 10, we show the effect of the shock on international trade. We observe a large drop in exports for the agriculture and food manufacturing industries. Exports primarily falls in these two industries for two reasons: i) they have the highest rate of price increases as seen in figure 9, ii) both industries have relatively high export elasticities. The increase in domestic prices also increase the import shares for both final consumption goods and inputs purchased by domestic industries. For final consumption, the results suggest that the largest substitution effect in the consumer basket occurs for meat consumption. For the consumption of inputs, industries that rely heavily on inputs from the agriculture and food manufacturing industries increase their import share as foreign-produced inputs become relatively cheaper. Accounting for the combined effects on both exports and imports, real net exports decline by approximately 5.5 percent.

Figure 10: Changes in exports and imports

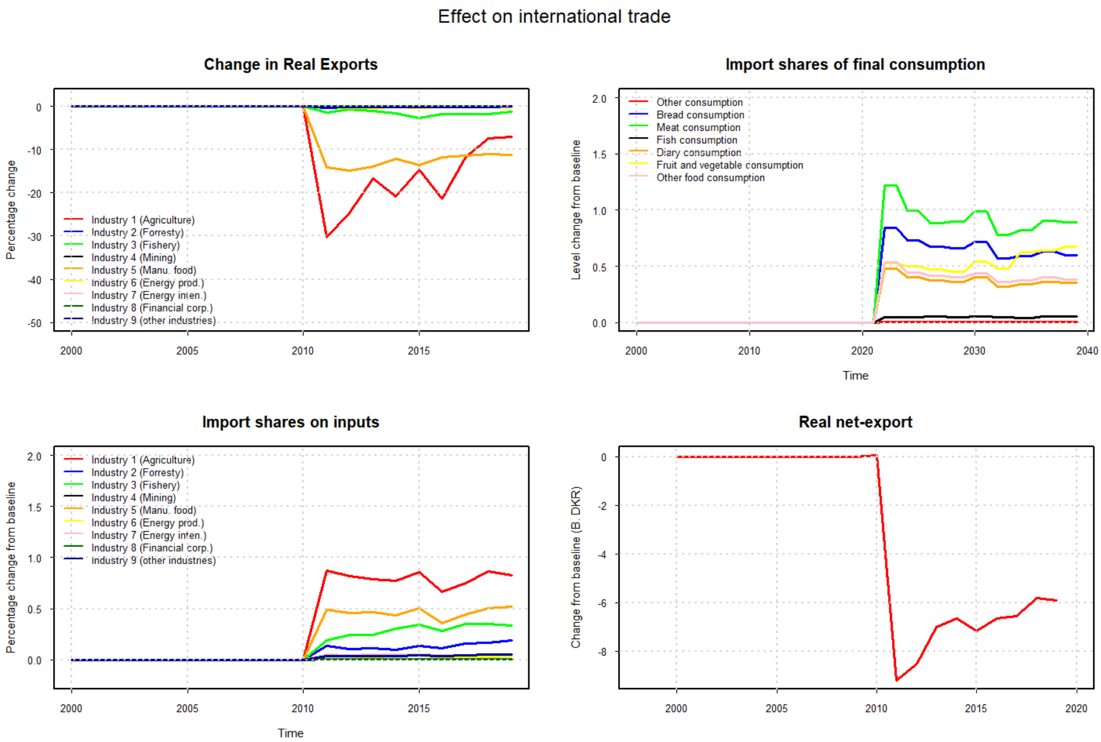
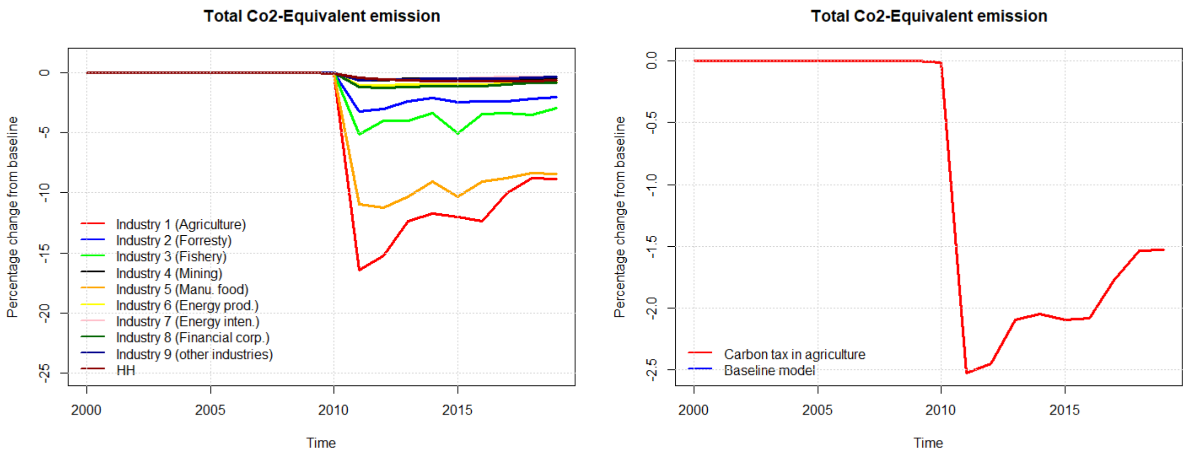


Figure 11 presents the impact of carbon taxes on CO2E emissions across individual industries and at the aggregate level. The findings indicate a reduction in CO2E emissions across all industries. Notably, emissions in the agricultural and food manufacturing sectors decrease by approximately 9% by the end of the simulation period. This decline is primarily driven by reduced production, as final demand—particularly exports—decreases for these industries. At the aggregate level, CO2E emissions decline by 1.5% in the final period of the simulation.

Figure 11: Changes in CO2E emissions



We now turn to a critical aspect of the carbon tax, focusing on which sectors bear the economic cost of emission reductions. Figure 12 illustrates the changes in financial net wealth across the non-financial, household, government, and rest of the world sectors. While the government benefits from increased tax revenues due to the carbon tax, it seems to bear the largest portion of the associated economic burden. Specifically, the revenue gained from carbon taxation is outweighed by revenue losses from reduced production. Financial net wealth declines in all domestic sectors, which, according to accounting identities, leads to a corresponding increase in net wealth for the rest of the world sector.

Figure 12: Changes in financial net wealth

The effects of a contractionary fiscal policy shock (in the form of a carbon tax), leading to a fall in economic activity and reduced emissions, are in line with economic intuition. In the model structure, the substitution effects between domestic and foreign-produced goods play a significant role. These core macroeconomic channels, along with stock-flow consistency, allow for an assessment of which sectors incur costs or benefit from this policy.

7. Conclusion

This paper developed an ecological macroeconomic model by combining the Stock-flow-Consistent approach (SFC) with an Input-Output (IO) framework. The model structure was divided into four blocks: i) the domestic production block, describing total production, production costs, and profits for the domestic industries, ii) the aggregate (or final) demand block, providing a detailed description of the drivers of aggregate demand, iii) the stock-flow-consistency block, in which we modelled the financial aspects of each institutional sector to capture the interdependence between real and financial spheres of the economy, while ensuring there are no leakages in the system, and finally iv) the environmental block, where we modelled the types of energies used in the production process while also capturing GHG emissions of each industry and the economy as a whole.

While building the model structure, most of the model parameters were estimated using annual time series data from 1995 to 2019. To assess model validity, we performed two standard checks. First, we numerically solved the model to establish a baseline, the results of which were compared with the observed data. We found that the model decently captured the overall tendency and fluctuation in key variables related to economic activity and environment. Second, we analysed the response of the model to a variety of shocks related to monetary and fiscal policy. We found that the model effectively captures the stylized facts and that these shocks affect the economy through multiple channels, which are important in policy making. We believe, our model in this paper will serve as a foundation, which can be extended in a variety of ways. The model has the potential to offer a reasonable assessment of the climate policies to the relevant stakeholders.

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8. Appendix:

8.1. Industry statistics

The industry level data imported from Statistics Denmark includes a total of 117 industries. In this paper, we aggregate these 117 industries to 9 to obtain a simpler representation of the economy. The 9 industries are presented below together with key environmental and economic statistics.[56]

Table A1: Industry statistics

	Statistics			
Industry	CO2E Emissions (Mio. tons)	CO2E Intensity (Mio. tons per Bil. DKK)	Export Share (pct)	Total Sales (Bil. DKK)
Agricultural	12.24	0.19	0.28	65.32
Forresty	0.06	0.02	0.08	3.60
Fishery	0.46	0.12	0.71	3.99
Mining	2.30	0.04	0.48	55.65
Manufacturing food	1.47	0.01	0.63	129.25
Energy supply and refinaries	29.18	0.32	0.34	90.80
Other energy intensive industries	6.84	0.03	0.67	219.32
Financial corp	0.07	0.00	0.05	160.25
Other industries	47.97	0.02	0.24	2392.41

Note:
Data for the year 2019. CO2 Intensity is calculated as CO2 Emissions divided by domestic production. Export share is calculated as exported sales divided by total sales

8.2. Distribution of gross operating surplus and mixed income (B2)

This appendix will provide a description on how we make the transition from industry to sectoral level in the model using gross operating surplus as the combining variable. The main reason that we need to make this transition is that not all entries below gross surplus in the TFM (table 4) are available at an industrial level.

The most appropriate point for transitioning from industry to sectoral accounts is at the gross operating surplus and mixed income (B2), as this is the final entry reflected in the input-output tables (see Table 2). The objective is to ensure a consistent transition by aligning observed data for B2 at both the industry and sectoral levels. A common practice in the Stock-Flow-Consistent literature is to assume that the non-financial corporations sector collects the entire gross operating surplus, which is then distributed to the household, government, and financial corporation sectors

[56] The "Other energy intensive" industry includes industries that are not part of the energy production and supply, but still regulated by the ETS program.

using exogenous shares. This method could be easily incorporated into the current model by summing the total gross operating surplus of mixed income across industries and then allocating it according to these sectoral shares. This approach assumes that all industries are equally linked to the four domestic sectors based on exogenous shares, which diminishes the value of including industrial aggregation in the model. For instance, consider the impact of a carbon tax on the agricultural industry, which reduces gross operating surplus. If we fail to account for the fact that this industry is predominantly "owned" by the household sector (67%) and instead use aggregate sectoral shares, where the household sector holds a weight of just 21%, we will underestimate the effect on the household balance sheet.

To calculate these shares at the industry level, we rely on the *industry by sector* matrix provided by Statistics Denmark (DST 2021), which details the industrial contributions to sectoral accounts based on gross value added (GVA) in 2016. This matrix helps us estimate how gross operating surplus and mixed income should be allocated among the four domestic sectors across the nine industries. The process involves the following steps:

1. Within each industry we calculate the weight of NFCs, FCs, households, and the government for each row in the matrix of *industry by sector*.
2. As the matrix of *industry by sector* is disaggregated into more than 90 industries, we use gross operating surplus and mixed income for each industry to make a weighted average for the shares at the 9-industry level used in this paper.[57]
3. As these shares are calculated based on 2016 data, they have most likely changed since 1998, in which we start the simulation of the model. To take this into account, we include a trend calculated from the sectoral to total gross operating surplus and mixed income and apply this trend on the industry level shares calculated in step 2.
4. Lastly, we calculate an adjustment term to consider discrepancies between our estimated sectoral gross operating surplus and mixed income variables, based on the first 3 steps, and observed gross operating surplus and mixed income for each sector.

These steps result in equation 57a-57d in which gross operating surplus and mixed income are calculated for the household, government, financial corporations, and non-financial corporations. In the table below, we show the estimated shares for 2016.

Table A2: Sectoral weights by industry

[57] In the current version of the model, we only use the 2016 values for B2 to make this aggregation.

Sectoral distribution weights

	Statistics			
Industry	**NFC weight**	**FC weight**	**GOV weight**	**HH weight**
Agricultural	0.33	0.00	0.00	0.67
Forresty	0.00	0.00	0.00	1.00
Fishery	1.00	0.00	0.00	0.00
Mining	1.00	0.00	0.00	0.00
Manufacturing food	0.94	0.00	0.00	0.06
Energy supply and refinaries	1.00	0.00	0.00	0.00
Energy intensive industries	1.00	0.00	0.00	0.00
Financial corp	0.00	1.00	0.00	0.00
Other industries	0.67	0.00	0.11	0.22

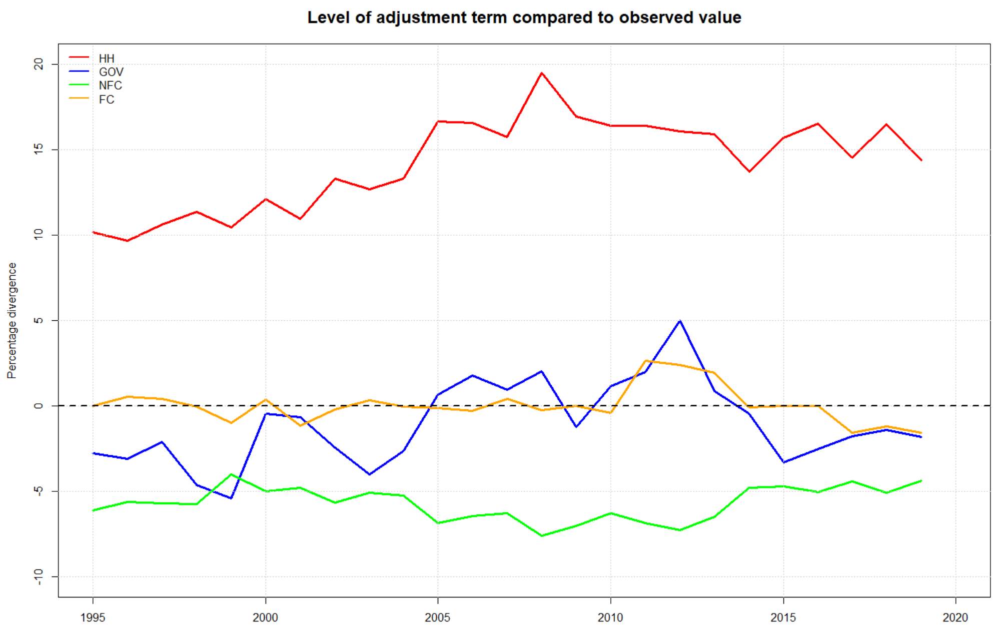
Note:
Own calculations

To give an example, for gross operating surplus in the agricultural industry, 67% will be associated with the household sector, while 33% will be associated with the non-financial corporation sector. For the Mining industry, gross operating surplus will be associated only with the non-financial corporation sector. As no gross operating surplus should be lost while making this transition, all rows sum to 1.

Lastly, in figure A1 we show the magnitude of the adjustment terms (calculated in step 4) relative to the observed value of gross operating surplus. The adjustment terms for the financial corporations and government sector are close to 0. The reason for the undershooting of gross operating surplus for the household sector (as the adjustment term is positive) and the overshooting of non-financial corporations gross operating surplus, are to be found in the matrix of *industry by sectors*.[58] A majority of industries show that household contribute by 0.0% to total gross operating surplus within a specific industry. When this notation is used, it means that the actual value is between 0.0% and 0.05%.[59]

[58] The level of adjustment in the household and non-financial corporation sector is almost equal in absolute values.
[59] In cases where the true value is 0%, the entry is empty.

Figure A1: Adjustment terms relative to observed values



8.3. Calculation of financial flows

In this appendix, we calculate three types of rates of returns:[60] i) first, we calculate 3 different interest rates (related to deposits, securities, loans, and other accounts), which will determine the net interest income denoted by *NINT,* ii) second, we calculate the rate of return on equity holdings, which determines the flow associated with equities (mostly dividends) denoted by *NDIV*, and iii) third, we calculate the rate of return on other investment income (related to pension and insurances), which determines the flow called *NOIR*. Note that the income flow related to reinvested earnings on FDI expresses the operating surplus of the foreign direct investment corporations. This is not related to any of the financial assets in the TFM.[61]

Following standard accounting, we know that payment flows in the current period depends on the stock of last period and the rate of returns from the last period. This relationship, expressed in equation A1, is used to calculate our rates of returns:

[60] We use unconsolidated data to calculate the rates of returns in the model.
[61] Rent is defined as income received by the owner of a natural resource and is simply included as a flow without associating it to any type of rate of return.

$$flow_t = rate_{t-1} * stock_{t-1} \Leftrightarrow rate_{t-1} = \frac{flow_t}{rate_{t-1}} \qquad \text{(equation A1)}$$

Focusing on interest rate calculations, first we calculate the interest rate on securities, assuming the rate of return on securities to be the same domestically and abroad. Therefore, we calculate the interest rate on securities (r_t^{SEC}) as the mean of the rate in Denmark and ECB as follows:

$$r_{t-1}^{SEC} = 0.5 * \ i\_sec_{t-1}^{DK} + 0.5 * \ i\_sec_{t-1}^{ECB} \qquad \text{(equation A2)}$$

By multiplying this rate of return with the stock of securities in the last period, we get the interest payment related to securities. The interest payment on securities is then subtracted from the overall interest payment, the resultant of which is used to calculate interest rate on deposits and loans.

To calculate the interest rate on deposits (r_t^{DEPO}) paid by the financial corporations, we take the total net interest *paid* by financial corporations, from which we subtract interest payments on securities *issued* by the financial corporations. The resultant is divided with the stock of deposits (representing a liability) for the financial corporations in the last period. This calculation can be expressed as follows:

$$r_{t-1}^{DEPO} = \frac{interest\ paid\ by\ FC_t - (r_{t-1}^{SEC} * secu_{t-1}^{FC,L})}{deposits\ in\ FC_{t-1}} \qquad \text{(equation A3a)}$$

where $r_{t-1}^{SEC} * secu_{t-1}^{FC,L}$ is the interest payments of financial corporations on their issued securities.

To calculate the interest rate on loans r_t^{LOA}, we use the same approach, i.e., we take the total net interest *received* by financial corporations, from which we subtract interest income received on securities *owned* by the financial corporations, which is then divided with the stock of loans (which represents an asset) issued by financial corporations in the last period. This is given by equation 8b as follows:

$$r_{t-1}^{LOA} = \frac{interest\ received\ by\ FC_t - (r_{t-1}^{SEC} * secu_{t-1}^{FC,A})}{Loan\ FC_{t-1}} \qquad \text{(equation A3b)}$$

where $r_{t-1}^{SEC} * secu_{t-1}^{FC,A}$ denotes interest payments received by financial corporations on holding securities as financial assets.

Once interest rates on interest bearing stocks are computed, we can show that the net interest payments (*NINT*) of sector s are given by the following equation:

$$NINT_t^S = r_t^{DEP} * (NDEPO_t^S + NDERV_t^S + NTCRED_t^S) + r_t^{SEC} * NSEC_t^S + r_t^{LOA} * NLOA_t^S \quad \text{(equation A4)}$$

Note that the rate of return on three financial stocks namely, net deposits (*NDEPO)*, net derivatives (*NDERV*), and net trade credits (*NTCRED*), is the same as denoted by interest rate r_t^{DEP}. The net stock of securities (*NSEC*) and net stock of loans (*NLOA*) are linked to their corresponding rate of returns denoted (r_t^{SEC}) and r_t^{LOA}, respectively.

The return on equities (r_t^{DIVD}) is also calculated using equation A1. This calculation is expressed in equation A5 as follows:

$$r_{t-1}^{DIVD} = \frac{dividends\ paid\ by\ FC_t}{equities\ issued\ by\ FC_{t-1}} \quad \text{(equation A5a)}$$

We can now determine net income received from dividend payments for sector s as follows:

$$NDIV_t^S = r_t^{DIVD} * NEQ_t^S \quad \text{(equation A5b)}$$

where *NEQ* represents the net stock of equities.

Similarly, the rate of return for insurance and pensions (r_t^{INSU}) is given by:

$$r_{t-1}^{INSU} = \frac{return\ paid\ by\ FC_t}{insurances\ issued\ by\ FC_{t-1}} \quad \text{(equation A6a)}$$

We can use the rate of return r_t^{INSU} to calculate the income associated with net insurance payments for sector s as follows:

$$NOIR_t^S = r_t^{INSU} * NINSU_t^S \quad \text{(equation A6b)}$$

where *NINSU* represents the net stock of insurance and pensions.

This concludes the presentation of industry and sectoral level variables used within the model. In the next section we instead focus on environmental data used in the model.

8.4. Substitution between consumption products

In this appendix, we show the two substitution effects modelled for final consumption. The two effects are i) substitution between product types, ii) substitution between domestically and foreign produced products.

Substitution between product types

To allow for substitution between product types, we endogenize the shares γ_t^{cp} (used in equation 19) following a similar method as used in the GreenREFORM and ADAM models, which includes a nested structure. In our model, the first nest includes a choice between industry specific consumption products (c_t^{spec}) and food products ($c_t^{110} + c_t^{120} + c_t^{130} + c_t^{140} + c_t^{160} + c_t^{180}$). In the second nest, consumers choose between the six different food products. The nested structure can be modelled as follows:[62]

$$\gamma_t^{c^{spec}} = \theta^{spec} * \left(\frac{ppcon_t^{oth,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(Equation A7.)}$$

$$\gamma_t^{c^{110}} = \theta^{110} * \left(\frac{ppcon_t^{110,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(Equation A8.)}$$

$$\gamma_t^{c^{120}} = \theta^{120} * \left(\frac{ppcon_t^{120,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(Equation A9.)}$$

$$\gamma_t^{c^{130}} = \theta^{130} * \left(\frac{ppcon_t^{130,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(Equation A10.)}$$

$$\gamma_t^{c^{140}} = \theta^{140} * \left(\frac{ppcon_t^{140,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(Equation A11.)}$$

$$\gamma_t^{c^{160}} = \theta^{160} * \left(\frac{ppcon_t^{160,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(Equation A12.)}$$

$$\gamma_t^{c^{180}} = \theta^{180} * \left(\frac{ppcon_t^{180,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(Equation A13.)}$$

Several parameters go into the equations above. Starting with σ^{nest2} and σ^{nest1} these represent the elasticity of substitution in the two nests. We set $\sigma^{nest2} = 0.8$ and $\sigma^{nest1} = 0.2$ which is slightly lower compared to the parameters used by GreenREFORM who use a different disaggregation of the consumer basket. Lastly, the parameters $\theta_t^{spec}, \theta^{110}, \theta^{120}, \theta^{130}, \theta^{140}, \theta^{160}$, and θ^{180} are calibrated to match the starting value of each corresponding share. Equation A.7-A.13 includes several price

[62] To ensure consistency, we model the share $\gamma_t^{c^{spec}}$ as a residual, to ensure that the sum of the shares always sum to 1.

deflators, all of them adjusted for the tax rate.[63] First, we define seven price deflators; one for each product type: [64]

$$ppcon_t^{p,tax} = \left(\frac{C_t^p}{\sum_{n=1}^{9} \frac{C_{dom,t}^{n\,p}}{py_t^n} + \sum_{n=1}^{9} \frac{C_{im,t}^{n\,p}}{pm_t^n} + \frac{C_{im,t}^{un\,p}}{pm_t^{un}}} \right) * \left(1 + tax_{rate,t}^p\right) \qquad \text{(Equation A14.)}$$

Thereby each of the 7 product types p are a function of the producer price indexes for the 9 domestic industries (py_t^n) as well as the ten foreign price indexes (pm_t^n, pm_t^{un}). As the consumer is faced with the price after taxes, we multiply on the average tax-rate for a given product ($tax_{rate,t}^p$). The average tax-rate is modelled as follows:

$$tax_{rate,t}^p = \frac{\left(C_{imd,t}^p + C_{ctax,t}^p + C_{VAT,t}^p\right)}{C_t^p} \qquad \text{(Equation A15.)}$$

Where $C_{imd,t}^p$ defines import duties, $C_{ctax,t}^p$ defines commodity taxes, and $C_{VAT,t}^p$ defines value added taxes all associated with consumption of product type p.

Besides from the 7 price indexes for each of the 7 product types ($ppcon_t^{p,tax}$), we need two aggregate price indexes, one for the entire consumption ($ppcon_t^{tax}$), and one for consumption of food products ($ppcon_t^{food,tax}$). We follow the same approach as in equation ($ppcon_t^{p,tax}$) but aggregate across the relevant product types p. The same goes for the average tax rate.

Thereby, we have the 9 price indexes for different consumption goods and baskets used in equation A7-A13. In the following we focus on the second substitution effect.

Substitution between domestic and foreign products

To allow substitution between domestic and foreign products, we need to calculate a domestic and foreign price index for each product type p. For each of the 7 product types in the consumption basket the price deflator is calculated as follows:

[63] We can use this tax-adjusted deflator to go from nominal value plus taxes to real-values without taxes. $c_{dom,t}^p * ppcon_t^{p,tax} = C_{dom,t}^p + C_{ctax,t}^p + C_{VAT,t}^p + C_{imd,t}^p$

[64] We include these tax rates within the consumer price indexes, as these rates are included within the final price paid by the consumer. Also, this allows us to implement carbon taxes on the consumers for different product types for future analysis.

$$ppcon^{p}_{dom,t} = \left(\frac{C^{tot,p}_{dom,t}}{\sum_{n=1}^{9} \frac{C^{n\,p}_{dom,t}}{py^{n}_{t}}} \right) \qquad \text{(Equation A16.)}$$

Where $C^{p}_{dom,t}$ is the nominal consumption of domestic goods of type *p*. The term in the denominator ($\sum_{n=1}^{9} \frac{C^{n,p}_{dom,t}}{py^{n}_{t}}$) is the real consumption for domestic goods of type *p,* calculated by taking the sum of nominal consumption in good type *p* for each industry *n* divided by the price deflator in industry *n*.

For the foreign produced products, we follow a similar approach:

$$ppcon^{p}_{im,t} = \left(\frac{C^{tot,p}_{im,t}}{\sum_{n=1}^{9} \frac{C^{n\,p}_{im,t}}{pm^{n}_{t}}} \right) \qquad \text{(Equation A17.)}$$

In this case, $C^{tot,p}_{im,t}$ is the nominal imported consumption of good type *p* and $\sum_{n=1}^{9} \frac{C^{n\,p}_{im,t}}{pm^{n}_{t}}$ represents the real imported consumption of good type *p*.

In equation A16 and A17, we do not account for the different tax rates as we did previously when modelling substitution between product types. The reason is that under the current WTO rules, domestically and imported goods in a specific market must be equally taxed. We therefore exclude the tax rate in both equations as it will play no role.

The two price indices can then be used to calculate the real exchange rate using equation 52, which is then used to calculate import shares in equation 51.

8.5. Industry aggregation:

IO117		IO69		ESFC-IO Denmark	
Code (117)	**Name (Danish)**	**Code (69)**	**Name (Danish)**	**Industry**	**Name**
10000	Landbrug og gartneri	1000	Landbrug og gartneri	1	Landbrug og Gartneri
20000	Skovbrug	2000	Skovbrug	2	Skovbrug
30000	Fiskeri	3000	Fiskeri	3	Fiskeri
60000 80090	Indvinding af olie og gas Indvinding af grus og sten	6090	Råstofindvinding	4	Råstofindvinding

90000	Service til råstofindvinding				
100010	Slagterier	10120	Føde-, drikke- og tobaksvareindustri	5	Manufacturing of food products
100020	Fiskeindustri				
100030	Mejerier				
100040	Bagerier, brødfabrikker mv.				
100050	Anden fødevareindustri				
110000	Drikkevareindustri				
120000	Tobaksindustri			9	Other industries
130000	Tekstilindustri	13150	Tekstil- og læderindustri		
140000	Beklædningsindustri				
150000	Læder- og fodtøjsindustri				
160000	Træindustri	16000	Træindustri	7	Other energy intensive industries
170000	Papirindustri	17000	Papirindustri		
180000	Trykkerier mv.	18000	Trykkerier mv.	9	Other industries
190000	Olieraffinaderier mv.	19000	Olieraffinaderier mv.	6	Energiforsyning og Raffinaderier
200010	Fremst. af basiskemikalier	20000	Kemisk industri	7	Other energy intensive industries
200020	Fremst. af maling og sæbe mv.				
210000	Medicinalindustri	21000	Medicinalindustri		
220000	Plast- og gummiindustri	22000	Plast- og gummiindustri	9	Other industries
230010	Glasindustri og keramisk industri	23000	Glas- og betonindustri	7	Other energy intensive industries
230020	Betonindustri og teglværker				
240000	Fremst. af metal	24000	Fremst. af metal		
250000	Metalvareindustri	25000	Metalvareindustri	9	Other industries
260010	Fremst. af computere og kommunikationsudstyr mv.	26000	Elektronikindustri		
260020	Fremst. af andet elektronisk udstyr				
270010	Fremst. af elektriske motorer mv.	27000	Fremst. af elektrisk udstyr		
270020	Fremst. af ledninger og kabler				
270030	Fremst. af husholdningsapparater, lamper mv.				
280010	Fremst. af motorer, vindmøller og pumper	28000	Maskinindustri	7	Other energy intensive industries
280020	Fremst. af andre maskiner			9	Other industries
290000	Fremst. af motorkøretøjer og dele hertil	29000	Fremst. af motorkøretøjer og dele hertil		
300000	Fremst. af skibe og andre transportmidler	30000	Fremst. af skibe og andre transportmidler		
310000	Møbelindustri	31320	Møbel- og anden industri		
320010	Fremst. af medicinske instrumenter mv.				

320020	Legetøj og anden fremstillingsvirksomhed				
330000	Reparation og installation af maskiner og udstyr	33000	Reparation og installation af maskiner og udstyr		
350010	Elforsyning	35000	Energiforsyning	6	Energiforsyning og Raffinaderier
350020	Gasforsyning				
350030	Varmeforsyning				
360000	Vandforsyning	36000	Vandforsyning	9	Other industries
370000	Kloak- og rensningsanlæg	37390	Renovation, affaldsbehandling mv.		
383900	Renovation, genbrug og forureningsbekæmpelse			7	Other energy intensive industries
410009	Nybyggeri	41430	Bygge og anlæg	9	Other industries
420000	Anlægsvirksomhed				
430003	Professionel reparation og vedligeholdelse af bygninger				
430004	Gør-det-selv reparation og vedligeholdelse af boliger				
450010	Bilhandel	45000	Bilhandel og -værksteder mv.		
450020	Bilværksteder mv.				
460000	Engroshandel	46000	Engroshandel		
470000	Detailhandel	47000	Detailhandel		
490010	Regional- og fjerntog	49000	Landtransport		
490020	Lokaltog, bus og taxi mv.				
490030	Fragtvognmænd og rørtransport				
500000	Skibsfart	50000	Skibsfart		
510000	Luftfart	51000	Luftfart		
520000	Hjælpevirksomhed til transport	52000	Hjælpevirksomhed til transport		
530000	Post og kurertjeneste	53000	Post og kurertjeneste		
550000	Hoteller mv.	55560	Hoteller og restauranter		
560000	Restauranter				
580010	Forlag	58000	Udgivervirksomhed		
580020	Udgivelse af computerspil og anden software				

590000	Produktion af film, tv og musik mv.	59600	Radio- og tv-stationer samt produktion af film, tv, mus		
600000	Radio- og tv-stationer				
610000	Telekommunikation	61000	Telekommunikation		
620000	It-konsulenter mv.	62630	It- og informationstjenester		
630000	Informationstjenester				
640010	Pengeinstitutter	64000	Finansiel virksomhed	8	Financial corporations
640020	Kreditforeninger mv.				
650000	Forsikring og pension	65000	Forsikring og pension		
660000	Finansiel service	66000	Finansiel service		
680010	Ejendomsmæglere mv.	68100	Ejendomsmæglere mv.	9	Other industries
680030	Udlejning af erhvervsejendomme	68300	Udlejning af erhvervsejendomme		
680023	Boliger, husleje i lejeboliger	68203	Boliger, husleje i lejeboliger		
680024	Boliger, ejerboliger mv.	68204	Boliger, ejerboliger mv.		
690010	Advokatvirksomhed	69700	Advokater, revisorer og virksomhedskonsulenter		
690020	Revision og bogføring				
700000	Virksomhedskonsulenter				
710000	Arkitekter og rådgivende ingeniører	71000	Arkitekter og rådgivende ingeniører		
720001	Forskning og udvikling, markedsmæssig	72001	Forskning og udvikling, markedsmæssig		
720002	Forskning og udvikling, ikke-markedsmæssig	72002	Forskning og udvikling, ikke-markedsmæssig		
730000	Reklame- og analysebureauer	73000	Reklame- og analysebureauer		
740000	Anden videnservice	74750	Dyrlæger og anden videnservice		
750000	Dyrlæger				
770000	Udlejning og leasing af materiel	77000	Udlejning og leasing af materiel		
780000	Arbejdsformidling og vikarbureauer	78000	Arbejdsformidling og vikarbureauer		
790000	Rejsebureauer	79000	Rejsebureauer		
800000	Vagt og sikkerhedstjeneste	80820	Rengøring og anden operationel service		
810000	Ejendomsservice, rengøring og anlægsgartnere				
820000	Anden operationel service				
840010	Offentlig administration	84202	Offentlig administration mv.		
840022	Forsvar, politi og retsvæsen mv., ikke-markedsmæssig				
840021	Redningskorps mv., markedsmæssig	84101	Redningskorps mv., markedsmæssig		
850010	Grundskoler	85202	Undervisning, ikke-markedsmæssig		
850020	Gymnasier og erhvervsfaglige skoler				
850030	Videregående uddannelsesinstitutioner				

850042	Voksenundervisning mv., ikke-markedsmæssig				
850041	Voksenundervisning mv., markedsmæssig	85101	Voksenundervisning mv., markedsmæssig		
860010	Hospitaler	86000	Sundhedsvæsen		
860020	Læger, tandlæger mv.				
870000	Plejehjem mv.	87880	Sociale institutioner		
880000	Daginstitutioner og dagcentre mv.				
900000	Teater, musik og kunst	90920	Kunst, kultur og spil		
910001	Biblioteker, museer mv., markedsmæssig				
910002	Biblioteker, museer mv., ikke-markedsmæssig				
920000	Lotteri og andet spil				
930011	Sport, markedsmæssig	93000	Sport, forlystelsesparker og andre fritidsaktiviteter		
930012	Sport, ikke-markedsmæssig				
930020	Forlystelsesparker og andre fritidsaktiviteter				
940000	Organisationer og foreninger	94000	Organisationer og foreninger		
950000	Reparation af husholdningsudstyr	95000	Reparation af husholdningsudstyr		
960000	Frisører, vaskerier og andre serviceydelser	96000	Frisører, vaskerier og andre serviceydelser		
970000	Private husholdninger med ansat medhjælp	97000	Private husholdninger med ansat medhjælp		

8.6. Model equations

I.) Industry level equations

Total Production, costs and profits

$$PROD^n_{dom,t} = \sum_{i=1}^{9} Z^{n\,i}_t + \sum_{p=1}^{7} C^{n\,p}_{dom,t} + GOV^n_{dom,t} + INV^n_{dom,t} + INVENT^n_{dom,t} + X^n_{dom,t} \quad \text{(M. 1)}$$

$$prod^n_{dom,t} = \sum_{i=1}^{9} z^{n\,i}_t + \sum_{p=1}^{7} c^{n\,p}_{dom} + gov^n_{dom,t} + inv^n_{dom,t} + invent^n_{dom,t} + x^n_{dom,t} \quad \text{(M.2)}$$

$$COST^n_{dom,t} = \sum_{i=1}^{9} Z^{i\,n}_t + Z^n_{uim,t} + Z^n_{imd,t} + CT^n_t + VAT^n_t + OPT^n_t + W^n_t \quad \text{(M.3)}$$

$$PROFIT^n_t = PROD^n_{dom,t} - COST^n_{dom,t} \quad \text{(M.4)}$$

Inputs in production, imports and industry taxes

$$z_t^{i\,n} = \ a_t^{i\,n} * prod_{dom,t}^n \tag{M.5}$$

$$z_{im,t}^{i\,n} = \phi_{z,t}^{i\,n} * z_t^{i\,n} \tag{M.6}$$

$$z_{dom,t}^{i\,n} = \left(1 - \phi_{z,t}^{i\,n}\right) * z_t^{i\,n} \tag{M.7}$$

$$\ln(\phi_{z,t}^{i\,n}) = \beta_0^{z^{i\,n}} + \beta_1^{z^n} * ln(rer_t^n) + adj_{\phi,t}^n \tag{M.8}$$

$$Z_{im,t}^{i\,n} = z_{im,t}^{i\,n} * pm_t^n \tag{M.9}$$

$$Z_{dom,t}^{i\,n} = z_{dom,t}^{i\,n} * py_t^n \tag{M.10}$$

$$z_{uim,t}^n = \gamma_{uim,t}^n * prod_{dom}^n \tag{M.11}$$

$$Z_{uim,t}^n = z_{uim,t}^n * pm_{uim} \tag{M.12}$$

$$CT_t^n = \left(\sum_{i=1}^{9} Z_{dom,t}^{i\,n}\right) * CT_{rate,t}^n \tag{M.13}$$

$$VAT_t^n = \left(\sum_{i=1}^{9} Z_{dom,t}^{i\,n}\right) * VAT_{rate,t}^n \tag{M.14}$$

$$OTP_{tax,t}^n = \ NE_{tax,t}^n + ENV_{tax,t}^n \tag{M.15}$$

$$NE_{tax,t}^n = \left(\sum_{i=1}^{9} Z_{dom,t}^{i\,n}\right) * NE_{rate,t}^n \tag{M.17}$$

$$M_{duty,t}^n = \left(\sum_{i=1}^{9} Z_{im,t}^{i\,n}\right) * M_{duty,rate,t}^n \tag{M.18}$$

$$M_{c,t}^{tot} = \sum_{n=1}^{9}\sum_{p=1}^{7} C_{im,t}^{n\,p} + \sum_{p=1}^{7} C_{uim,t}^p \tag{M.19}$$

$$M_{inv,t}^{tot} = \sum_{n=1}^{9} INV_{im,t}^n + \ INV_{uim,t} \tag{M.20}$$

$$M_{invent,t}^{tot} = \sum_{n=1}^{9} INVENT_{im,t}^n + \ INVENT_{uim,t} \tag{M.21}$$

$$M_{gov}^{tot} = \sum_{n=1}^{9} GOV_{im}^n + GOV_{uim,t} \tag{M.22}$$

$$M_x^{tot} = \sum_{n=1}^{9} X_{m,t}^n + \ X_{uim,t} \tag{M.23}$$

Final demand components

$$\ln\Delta(c_t^{tot}) = 0.37^{**}\, ln\Delta yd_t^H - 0.30^{***}\, ln\, c_{t-1}^{tot} + 0.29^{***} \ln yd_{t-1}^H + 0.01 \ln fnw_{t-1}^H - 0.002^{**}\, Trend \qquad \text{(M.24)}$$

$$c_t^p = \gamma_t^{c^p} * c_t^{tot} \qquad \text{(M.25)}$$

$$\gamma_t^{c^{spec}} = \theta^{oth} * \left(\frac{ppcon_t^{oth,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(M.26)}$$

$$\gamma_t^{c^{110}} = \theta^{110} * \left(\frac{ppcon_t^{110,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(M.27)}$$

$$\gamma_t^{c^{120}} = \theta^{120} * \left(\frac{ppcon_t^{120,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(M.28)}$$

$$\gamma_t^{c^{130}} = \theta^{130} * \left(\frac{ppcon_t^{130,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(M.29)}$$

$$\gamma_t^{c^{140}} = \theta^{140} * \left(\frac{ppcon_t^{140,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(M.30)}$$

$$\gamma_t^{c^{160}} = \theta^{160} * \left(\frac{ppcon_t^{160,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(M.31)}$$

$$\gamma_t^{c^{180}} = \theta^{180} * \left(\frac{ppcon_t^{180,tax}}{ppcon_t^{food,tax}}\right)^{\sigma^{nest2}} * \left(\frac{ppcon_t^{food,tax}}{ppcon_t^{tax}}\right)^{\sigma^{nest1}} \qquad \text{(M.32)}$$

$$c_{dom,t}^p = (1 - \phi_{c,t}^p) * c_t^p \qquad \text{(M.33)}$$

$$c_{im,t}^p = (\phi_{c,t}^p) * c_t^p \qquad \text{(M.34)}$$

$$\ln(\phi_{c,t}^p) = \beta_0^{c^p} + \beta_1^{c^p} * \ln(rer_t^p) + adj_{\phi,t}^p \qquad \text{(M.35)}$$

$$c_{dom,t}^{n\,p} = \lambda_{dom,t}^{n\,p} * c_{dom,t}^p \qquad \text{(M.36)}$$

$$c_{im,t}^{n\,p} = \lambda_{im,t}^{n\,p} * c_{im,t}^p \qquad \text{(M.37)}$$

$$c_{uim,t}^p = \gamma_{uim,t}^p * c_{im,t}^p \qquad \text{(M.38)}$$

$$C_{dom,t}^{n\,p} = c_{dom,t}^{n\,p} * py_t^n \qquad \text{(M.39)}$$

$$C_{im,t}^{n\,p} = c_{im,t}^{n\,p} * pm_t^n \qquad \text{(M.40)}$$

$$C_{uim,t}^p = c_{uim,t}^p * pm_t^{un}$$

$$C_{dom,t}^{tot,p} = \sum_{n=1}^{9} C_{dom,t}^{n\,p} \tag{M.41}$$

$$C_{im,t}^{tot,p} = \sum_{n=1}^{9} C_{im,t}^{n\,p} + C_{im,t}^{un\,p} \tag{M.42}$$

$$c_{dom,t}^{tot,p} = \sum_{n=1}^{9} c_{dom,t}^{n\,p} \tag{M.43}$$

$$c_{im,t}^{tot,p} = \sum_{n=1}^{9} c_{im,t}^{n\,p} + c_{im,t}^{un\,p} \tag{M.44}$$

$$C_{dom,t}^{tot,n} = \sum_{p=1}^{7} C_{dom,t}^{n\,p} \tag{M.45}$$

$$C_{im,t}^{tot,n} = \sum_{p=1}^{7} C_{im,t}^{n\,p} \tag{M.46}$$

$$C_{im,t}^{tot,un} = \sum_{p=1}^{7} C_{im,t}^{un\,p} \tag{M.47}$$

$$c_{dom,t}^{tot,n} = \sum_{p=1}^{7} c_{dom,t}^{n\,p} \tag{M.48}$$

$$c_{im,t}^{tot,n} = \sum_{p=1}^{7} c_{im,t}^{n\,p} \tag{M.49}$$

$$c_{im,t}^{tot,un} = \sum_{p=1}^{7} c_{im,t}^{un\,p} \tag{M.50}$$

$$c_{dom,t}^{tot} = \sum_{n=1}^{9} c_{dom,t}^{tot,n} \tag{M.51}$$

$$c_{im,t}^{tot} = \sum_{n=1}^{9} c_{im,t}^{tot,n} \tag{M.52}$$

$$C_{dom,t}^{tot} = \sum_{n=1}^{9} C_{dom,t}^{tot,n} \tag{M.53}$$

$$C_{im,t}^{tot} = \sum_{n=1}^{9} C_{im,t}^{tot,n} \tag{M.54}$$

$$C_{im,t}^{tot} = \sum_{n=1}^{9} C_{im,t}^{tot,n} \tag{M.55}$$

$$inv_t^{tot} = 1.8^{***} \Delta \ln\left(\frac{y_{t-1}}{k_{t-1}^{NFC}}\right) - 0.09^{***} \ln(inv_{t-1}^{tot}) + 1.42^{***}\, ps_{t-1} \tag{M.56}$$

$$inv_t^n = \lambda_{inv,t}^n * inv_t^{tot} \tag{M.57}$$

$$inv_{uim,t} = \gamma_{uim,t}^{inv} * inv_t^{tot}$$

$$inv_{dom,t}^n = \left(1 - \phi_{inv,t}^n\right) * inv_t^n \tag{M.58}$$

$$inv_{im,t}^n = \left(\phi_{inv,t}^n\right) * inv_t^n \tag{M.59}$$

$$INV_{dom,t}^n = inv_{dom,t}^n * py_t^n \tag{M.60}$$

$$INV_{im,t}^n = inv_{im,t}^n * pm_t^n \tag{M.61}$$

$$INV_{uim,t} = inv_{uim,t}^p * pm_t^{un} \tag{M.62}$$

$$inv_{dom,t}^{tot} = \sum_{n=1}^{9} inv_{dom,t}^n \tag{M.63}$$

$$inv_{im,t}^{tot} = \sum_{n=1}^{9} inv_{im,t}^n \tag{M.64}$$

$$INV_{dom,t}^{tot} = \sum_{n=1}^{9} INV_{dom,t}^n \tag{M.65}$$

$$INV_{im,t}^{tot} = \sum_{n=1}^{9} INV_{im,t}^n \tag{M.66}$$

$$\ln\left(\frac{x_t^n}{m_t^{n*}}\right) = \alpha_0^n + \alpha_1^n * \ln(rer_{t-1}^n) + adj_{x,t}^n \tag{M.67}$$

$$x_{dom,t}^n = \left(1 - \phi_{x,t}^n\right) * x_t^n \tag{M.68}$$

$$x_{m,t}^n = \left(\phi_{x,t}^n\right) * x_t^n \tag{M.69}$$

$$X_{dom,t}^n = x_{dom,t}^n * py_t^n \tag{M.70}$$

$$X_{m,t}^n = x_{m,t}^n * pm_t^n \tag{M.71}$$

$$x_{dom,t}^{tot} = \sum_{n=1}^{9} x_{dom,t}^n \tag{M.72}$$

$$x_{im,t}^{tot} = \sum_{n=1}^{9} x_{im,t}^n \tag{M.73}$$

$$X_{dom,t}^{tot} = \sum_{n=1}^{9} X_{dom,t}^{n} \tag{M.74}$$

$$X_{m,t}^{tot} = \sum_{n=1}^{9} X_{m,t}^{n} \tag{M.75}$$

Labor market and prices

$$py_t^n = (1 + \mu_t^n) * \frac{COST_{dom,t}^n}{prod_{dom,t}^n} \tag{M.76}$$

$$rer_t^n = xr_t * \frac{py_t^n}{pm_t^n} \tag{M.77}$$

$$ppcon_t^{p,tax} = \left(\frac{C_t^p}{\sum_{n=1}^{9} \frac{C_{dom,t}^{n\,p}}{py_t^n} + \sum_{n=1}^{9} \frac{C_{im,t}^{n\,p}}{pm_t^n} + \frac{C_{im,t}^{un\,p}}{pm_t^{un}}} \right) * \left(1 + tax_{rate,t}^p\right) \tag{M.78}$$

$$tax_{rate,t}^p = \frac{\left(C_{imd,t}^p + C_{ctax,t}^p + C_{VAT,t}^p\right)}{C_t^p} \tag{M.79}$$

$$ppcon_t^p = \left(\frac{C_{dom,t}^{tot,p}}{\sum_{n=1}^{9} \frac{C_{dom,t}^{n\,p}}{py_t^n}} \right) \tag{M.80}$$

$$pm_t^p = \left(\frac{C_{im,t}^{tot,p}}{\sum_{n=1}^{9} \frac{C_{im,t}^{n\,p}}{pm_t^n}} \right) \tag{M.81}$$

$$rer_t^p = xr_t * \frac{ppcon_t^p}{pm_t^p} \tag{M.82}$$

$$EMP_t^n = \frac{prod_t^n}{a_t^n} \tag{M.83}$$

$$UNEMP_t = LF_t - \sum_{n=1}^{9} EMP_t^n \tag{M.84}$$

$$UR_t = \frac{UNEMP_t}{LF_t} \tag{M.85}$$

$$W_t^n = Wage_t^n * EMP_t^n \tag{M.86}$$

$$ln\Delta\left(Wage_t^{gen}\right) = 0.24^{***} + 0.78^{***} ln\Delta\left(Wage_{t-2}^{gen}\right) + 0.08^{**} ln\Delta\left(Wage_{t-2}^{gen,T}\right) - 0.15^{**} \ln\left(Wage_{t-1}^{gen}\right) + 0.09^{*} \ln\left(Wage_{t-2}^{gen,T}\right) + 0.28^{***} \ln(a_{t-1}) \quad (M.87)$$

$$Wage_t^{gen,T} = Wage_{t-1}^{gen} * (1 + \pi_{t-1}) \quad (M.88)$$

$$\ln\left(Wage_t^n\right) = \omega_0 + \omega_1 \ln\left(Wage_t^{gen}\right) \quad (M.89)$$

II.) Sectoral level equations

Non-financial corporations

$$B2_t^{agg} = \sum_{n=1}^{9} PROFIT_t^n \quad (M.90)$$

$$B2^{NFC} = B2_t^{agg} - (B2^H + B2^{FC} + B2^G) \quad (M.91)$$

$$I_t^{NFC} = I_t^{agg} - (I_t^H + I_t^{FC} + I_t^G) \quad (M.92)$$

$$Y_t^{NFC} = Y_t - (B2_t^{agg} - B2_t^{NFC}) - \left(NTax_{prod,t} + OPT_t^{tot}\right) - W_t^{NFC} + NINT_t^{NFC} + NDIV_t^{NFC} + NOIR_t^{NFC} + NREFDI_t^{NFC} + OCT_t^{NFC} \quad (M.93)$$

$$YD_t^{NFC} = Y_t^{NFC} - ITAX_t^{NFC} \quad (M.94)$$

$$S_t^{NFC} = YD_t^{NFC} \quad (M.95)$$

$$NL_t^{NFC} = S_t^{NFC} - I_t^{NFC} - INVENT_t^{NFC} - NP_t^{NFC} - CT_t^{NFC} \quad (M.96)$$

$$FNL_t^{NFC} = NDEPO_{tr,t}^{NFC} + NSEC_{tr,t}^{NFC} + NLOA_{tr,t}^{NFC} + NEQ_{tr,t}^{NFC} + NINSU_{tr,t}^{NFC} + NDERV_{tr,t}^{NFC} + NTCRED_{tr,t}^{NFC} \quad (M.97)$$

$$FNW_t^{NFC} = NDEPO_t^{NFC} + NSEC_t^{NFC} + NLOA_t^{NFC} + NEQ_t^{NFC} + NINSU_t^{NFC} + NDERV_t^{NFC} + NTCRED_t^{NFC} \quad (M.98)$$

$$\left(\frac{NLOA_t^{NFC}}{K_t^{NFC}}\right) = 0.26^{***} + 0.28^{**}\left(\frac{I_{t-1}^{NFC}}{S_{t-1}^{NFC}}\right) - 2.11^{***} r_{t-1}^{LOA} \quad (M.99)$$

$$NLOA_{tr,t}^{NFC} = \Delta NLOA_t^{NFC} - NLOA_{rv,t}^{NFC} \quad (M.100)$$

$$NSEC_t^{NFC} = NSEC_{t-1}^{NFC} + NSEC_{tr,t}^{NFC} + NSEC_{rv,t}^{NFC} \quad (M.101)$$

$$NINSU_t^{NFC} = NINSU_{t-1}^{NFC} + NINSU_{tr,t}^{NFC} + NINSU_{rv,t}^{NFC} \quad (M.102)$$

$$NDERV_t^{NFC} = NDERV_{t-1}^{NFC} + NDERV_{tr,t}^{NFC} + NDERV_{rv,t}^{NFC} \quad (M.103)$$

$$NTCRED_t^{NFC} = NTCRED_{t-1}^{NFC} + NTCRED_{tr,t}^{NFC} + NTCRED_{rv,t}^{NFC} \quad (M.104)$$

$$NEQ_{tr,t}^{NFC} = -\left(NEQ_{tr,t}^{FC} + NEQ_{tr,t}^{H} + NEQ_{tr,t}^{G} + NEQ_{tr,t}^{ROW}\right) \quad (M.105)$$

$$NEQ_t^{NFC} = NEQ_{t-1}^{NFC} + NEQ_{tr,t}^{NFC} + NEQ_{rv,t}^{NFC} \quad (M.106)$$

$$NDEPO_{tr,t}^{NFC} = NL_t^{NFC} + NL_{adj,t}^{NFC} - (NSEC_{tr,t}^{NFC} + NLOA_{tr,t}^{NFC} + NINSU_{tr,t}^{NFC} + NDERV_{tr,t}^{NFC} + NTCRED_{tr,t}^{NFC} + NEQ_{tr,t}^{NFC}) \tag{M.107}$$

$$NDEPO_t^{NFC} = NDEPO_{t-1}^{NFC} + NDEPO_{tr,t}^{NFC} + NDEPO_{rv,t}^{NFC} \tag{M.108}$$

$$NINT_t^{NFC} = r_t^{DEP} * (NDEPO_t^{NFC} + NDERV_t^{NFC} + NTCRED_t^{NFC}) + r_t^{SEC} * NSEC_t^{NFC} + r_t^{LOA} * NLOA_t^{NFC} \tag{M.109}$$

$$NDIV_t^{NFC} = r_t^{DIVD} * NEQ_t^{NFC} \tag{M.110}$$

$$NOIR_t^{NFC} = r_t^{INSU} * NINSU_t^{NFC} \tag{M.111}$$

Households

$$B2_t^H = \sum_{n=1}^{9} PROFIT_t^n * S_{H,t}^{profit,n} + \mathrm{adj}_\mathrm{t}^\mathrm{H} \tag{M.112}$$

$$I_t^H = \sum_{n=1}^{9} INV_t^{agg} * S_{H,t}^{inv} \tag{M.113}$$

$$W_t^{NFC} = \sum_{n=1}^{9} W_t^n \tag{M.114}$$

$$W_t^H = W_t^{NFC} - W_t^{ROW} \tag{M.115}$$

$$Y_t^H = B2_t^H + W_t^H + NINT_t^H + NDIV_t^H + NOIR_t^H + NREFDI_t^H - SCON_{p,t}^H + SBEN_{r,t}^H + OCT_t^H \tag{M.116}$$

$$YD_t^H = Y_t^H - ITAX_t^H \tag{M.117}$$

$$S_t^H = YD_t^H - C_t^{agg} + PEN_t^{adj} \tag{M.118}$$

$$NL_t^H = S_t^H - I_t^H - INVENT_t^H - NP_t^H - CT_t^H \tag{M.119}$$

$$FNL_t^H = NDEPO_{t,tr}^H + NSEC_{t,tr}^H + NLOA_{t,tr}^H + NEQ_{t,tr}^H + NINSU_{t,tr}^H + NDERV_{t,tr}^H + NTCRED_{t,tr}^H \tag{M.120}$$

$$FNW_t^H = NDEPO_t^H + NSEC_t^H + NLOA_t^H + NEQ_t^H + NINSU_t^H + NDERV_t^H + NTCRED_t^H \tag{M.121}$$

$$EQ_{ratio,t}^H = \left(\frac{NEQ_t^H - NEQ_{rv,t}^H}{FNW_{t-1}^H}\right) \tag{M.122}$$

$$\Delta EQ_{ratio,t}^H = 0.33^* \Delta EQ_{ratio,t-1}^H + 0.060\Delta \frac{\left(NDIV_{t-1}^H + NEQ_{rv,t-1}^H\right)}{NEQ_{t-2}^H} - 0.18 EQ_{ratio,t-1}^H + 0.10^* \frac{\left(NDIV_{t-2}^H + NEQ_{rv,t-2}^H\right)}{NEQ_{t-3}^H} \tag{M.123}$$

$$NEQ_t^H = EQ_{ratio,t}^H * FNW_{t-1}^H + NEQ_{rv,t}^H \tag{M.124}$$

$$NEQ^{H}_{tr,t} = \Delta NEQ^{H}_{t} - NEQ^{H}_{rv,t} \quad \text{(M.125)}$$

$$LOA^{H}_{ratio,t} = \left(-\frac{NLOA^{H}_{t}}{YD^{H}_{t}}\right) \quad \text{(M.126)}$$

$$LOA^{H}_{ratio,t} = 0.91^{***}LOA_{ratio,t-1} + 2.28^{***}\left(\frac{I^{H}_{t}}{YD^{H}_{t}}\right) - 0.31r^{LOA}_{t} - 0.37^{***}D_{2016} \quad \text{(M.127)}$$

$$NLOA^{H}_{t} = -LOA^{H}_{ratio,t} * YD^{H}_{t} \quad \text{(M.128)}$$

$$NLOA^{H}_{tr,t} = \Delta NLOA^{H}_{t} - NLOA^{H}_{rv,t} \quad \text{(M.129)}$$

$$NDEPO^{H}_{tr,t} = NL^{H}_{t} + NL^{H}_{adj,t} - (NSEC^{H}_{tr,t} + NLOA^{H}_{tr,t} + NINSU^{H}_{tr,t} + NDERV^{H}_{tr,t} + NTCRED^{H}_{tr,t} + NEQ^{H}_{tr,t}) \quad \text{(M.130)}$$

$$NDEPO^{H}_{t} = NDEPO^{H}_{t-1} + NDEPO^{H}_{tr,t} + NDEPO^{H}_{rv,t} \quad \text{(M.131)}$$

$$NSEC^{H}_{t} = NSEC^{H}_{t-1} + NSEC^{H}_{tr,t} + NSEC^{H}_{rv,t} \quad \text{(M.132)}$$

$$NINSU^{H}_{t} = NINSU^{H}_{t-1} + NINSU^{H}_{tr,t} + NINSU^{H}_{rv,t} \quad \text{(M.133)}$$

$$NDERV^{H}_{t} = NDERV^{H}_{t-1} + NDERV^{H}_{tr,t} + NDERV^{H}_{rv,t} \quad \text{(M.134)}$$

$$NTCRED^{H}_{t} = NTCRED^{H}_{t-1} + NTCRED^{H}_{tr,t} + NTCRED^{H}_{rv,t} \quad \text{(M.135)}$$

$$NINT^{H}_{t} = r^{DEP}_{t} * (NDEPO^{H}_{t} + NDERV^{H}_{t} + NTCRED^{H}_{t}) + r^{SEC}_{t} * NSEC^{H}_{t} + r^{LOA}_{t} * NLOA^{H}_{t} \quad \text{(M.136)}$$

$$NDIV^{H}_{t} = r^{DIVD}_{t} * NEQ^{H}_{t} \quad \text{(M.137)}$$

$$NOIR^{H}_{t} = r^{INSU}_{t} * NINSU^{H}_{t} \quad \text{(M.138)}$$

Financial corporations

$$B2^{FC}_{t} = \sum_{n=1}^{9} PROFIT^{n}_{t} * S^{profit,n}_{FC,t} + \mathrm{adj}^{\mathrm{FC}}_{\mathrm{t}} \quad \text{(M.139)(M.135)}$$

$$I^{FC}_{t} = \sum_{n=1}^{9} INV^{agg}_{t} * S^{inv}_{FC,t} \quad \text{(M.140)}$$

$$Y^{FC}_{t} = B2^{FC}_{t} + NINT^{FC}_{t} + NDIV^{FC}_{t} + NOIR^{FC}_{t} + NREFDI^{FC}_{t} + SCON^{FC}_{r,t} - SBEN^{FC}_{p,t} + OCT^{FC}_{t} \quad \text{(M.141)}$$

$$YD^{FC}_{t} = Y^{FC}_{t} - ITAX^{FC}_{t} \quad \text{(M.142)}$$

$$S^{FC}_{t} = YD^{FC} - PEN^{adj}_{t} \quad \text{(M.143)}$$

$$NL^{FC}_{t} = S^{FC}_{t} - I^{FC}_{t} - INVENT^{FC}_{t} - NP^{FC}_{t} - CT^{FC}_{t} \quad \text{(M.144)}$$

$$FNL^{FC}_{t} = NGOLD^{FC}_{t,tr} + NDEPO^{FC}_{t,tr} + NSEC^{FC}_{t,tr} + NLOA^{FC}_{t,tr} + NEQ^{FC}_{t,tr} + NINSU^{FC}_{t,tr} + NDERV^{FC}_{t,tr} + NTCRED^{FC}_{t,tr} \quad \text{(M.145)}$$

$$FNW_t^{FC} = NGOLD_t^{FC} + NDEPO_t^{FC} + NSEC_t^{FC} + NLOA_t^{FC} + NEQ_t^{FC} + NINSU_t^{FC} + NDERV_t^{FC} + NTCRED_t^{FC} \quad \text{(M.146)}$$

$$NDEPO_{tr,t}^{FC} = -\left(NDEPO_{tr,t}^{NFC} + NDEPO_{tr,t}^{H} + NDEPO_{tr,t}^{G} + NDEPO_{tr,t}^{ROW}\right) \quad \text{(M.147)}$$

$$NLOA_{tr,t}^{FC} = -\left(NLOA_{tr,t}^{NFC} + NLOA_{tr,t}^{H} + NLOA_{tr,t}^{G} + NLOA_{tr,t}^{ROW}\right) \quad \text{(M.148)}$$

$$NINSU_{tr,t}^{FC} = -\left(NINSU_{tr,t}^{NFC} + NINSU_{tr,t}^{H} + NINSU_{tr,t}^{G} + NINSU_{tr,t}^{ROW}\right) \quad \text{(M.149)}$$

$$NDERV_{tr,t}^{FC} = -\left(NDERV_{tr,t}^{NFC} + NDERV_{tr,t}^{H} + NDERV_{tr,t}^{G} + NDERV_{tr,t}^{ROW}\right) \quad \text{(M.150)}$$

$$NDEPO_t^{FC} = NDEPO_{t-1}^{FC} + NDEPO_{tr,t}^{FC} + NDEPO_{rv,t}^{FC} \quad \text{(M.151)}$$

$$NLOA_t^{FC} = NLOA_{t-1}^{FC} + NLOA_{tr,t}^{FC} + NLOA_{rv,t}^{FC} \quad \text{(M.152)}$$

$$NINSU_t^{FC} = NINSU_{t-1}^{FC} + NINSU_{tr,t}^{FC} + NINSU_{rv,t}^{FC} \quad \text{(M.153)}$$

$$NDERV_t^{FC} = NDERV_{t-1}^{FC} + NDERV_{tr,t}^{FC} + NDERV_{rv,t}^{FC} \quad \text{(M.154)}$$

$$NSEC_{tr,t}^{FC} = NL_t^{FC} + NL_{adj,t}^{FC} - (NGOLD_{t,tr}^{FC} + NDEPO_{tr,t}^{FC} + NLOA_{tr,t}^{FC} + NINSU_{tr,t}^{FC} + NDIR_{tr,t}^{FC} + NCRED_{tr,t}^{FC} + NEQ_{tr,t}^{FC}) \quad \text{(M.155)}$$

$$NSEC_t^{FC} = NSEC_{t-1}^{FC} + NSEC_{tr,t}^{FC} + NSEC_{rv,t}^{FC} \quad \text{(M.156)}$$

$$NGOLD_t^{FC} = NGOLD_{t-1}^{FC} + NGOLD_{tr,t}^{FC} + NGOLD_{rv,t}^{FC} \quad \text{(M.157)}$$

$$NTCRED_t^{FC} = NTCRED_{t-1}^{FC} + NTCRED_{tr,t}^{FC} + NTCRED_{rv,t}^{FC} \quad \text{(M.158)}$$

$$NINT_t^{FC} = r_t^{DEP} * (NDEPO_t^{FC} + NDERV_t^{FC} + NTCRED_t^{FC}) + r_t^{SEC} * NSEC_t^{FC} + r_t^{LOA} * NLOA_t^{FC} \quad \text{(M.159)}$$

$$NDIV_t^{FC} = r_t^{DIVD} * NEQ_t^{FC} \quad \text{(M.160)}$$

$$NOIR_t^{FC} = r_t^{INSU} * NINSU_t^{FC} \quad \text{(M.161)}$$

Government

$$B2_t^G = \sum_{n=1}^{9} PROFIT_t^n * S_{G,t}^{profit,n} + \mathrm{adj}_t^G \quad \text{(M.162)}$$

$$I_t^{GOV} = \sum_{n=1}^{9} INV_t^{agg} * S_{GOV,t}^{inv} \quad \text{(M.163)}$$

$$CT_t^{tot} = \sum_{n=1}^{9} CT_t^n + C_{ctax,t}^{tot} + GOV_{ctax,t} + INV_{ctax,t}^{tot} + INVENT_{ctax,t}^{tot} + X_{ctax,t}^{tot} \quad \text{(M.164)}$$

$$VAT_t^{tot} = \sum_{n=1}^{9} VAT_t^n + C_{VAT,t}^{tot} + GOV_{VAT,t}^{tot} + INV_{VAT,t}^{tot} + INVENT_{VAT,t}^{tot} + X_{VAT,t}^{tot} \tag{M.165}$$

$$OPT_t^{tot} = \sum_{n=1}^{9} OTP_{tax,t}^n \tag{M.166}$$

$$M_{duty,t}^{tot} = \sum_{n=1}^{9} M_{duty,t}^n + M_{duty,t}^C + M_{duty,t}^{GOV} + M_{duty,t}^{INV} + M_{duty,t}^X \tag{M.167}$$

$$NTAX_t^{prod} = CT_t^{tot} + VAT_t^{tot} + M_{duty,t}^{tot} \tag{M.168}$$

$$IMTAX_{p,t} = M_{duty,t}^{tot} + OIMTAX_p \tag{M.169}$$

$$NIMTax_t = IMTAX_r - IMTAX_p \tag{M.170}$$

$$ITAX_t^H = 0.36 * Y_t^H \tag{M.171}$$

$$ITAX_t^{NFC} = 0.12 * Y_t^{NFC} \tag{M.172}$$

$$ITAX_t^{FC} = 0.08 * Y_t^{FC} \tag{M.173}$$

$$ITAX_t^{tot} = ITAX_t^H + ITAX_t^{NFC} + ITAX_t^{FC} + NITAX_t^{ROW} \tag{M.174}$$

$$YD_t^{GOV} = B2_t^{GOV} + NTAX_t^{prod} + OPT_t^{prod} + NIMTax_t + ITAX_t^{GOV,r} + NINT_t^{GOV} + NDIV_t^{GOV} + NOIR_t^{GOV} + NREFDI_t^{GOV} + SCON_{r,t}^{GOV} - SBEN_{p,t}^{GOV} + OCT_t^{GOV} \tag{M.175}$$

$$S_t^{GOV} = YD_t^{GOV} - G_t^{agg} \tag{M.176}$$

$$NL_t^G = S_t^{GOV} - I_t^{GOV} - INVENT_t^{GOV} - NP_t^{GOV} - CT_t^{GOV} \tag{M.177}$$

$$FNL_t^{GOV} = NDEPO_{t,tr}^{GOV} + NSEC_{t,tr}^{GOV} + NLOA_{t,tr}^{GOV} + NEQ_{t,tr}^{GOV} + NINSU_{t,tr}^{GOV} + NDERV_{t,tr}^{GOV} + NTCRED_{t,tr}^{GOV} \tag{M.178}$$

$$FNW_t^{GOV} = NDEPO_t^{GOV} + NSEC_t^{GOV} + NLOA_t^{GOV} + NEQ_t^{GOV} + NINSU_t^{GOV} + NDERV_t^{GOV} + NTCRED_t^{GOV} \tag{M.179}$$

$$NSEC_{tr,t}^G = NL_t^G + NL_{adj,t}^G - (NGOLD_{t,tr}^G + NDEPO_{tr,t}^G + NLOA_{tr,t}^G + NINSU_{tr,t}^G + NDIR_{tr,t}^G + NCRED_{tr,t}^G + NEQ_{tr,t}^G) \tag{M.180}$$

$$NSEC_t^G = NSEC_{t-1}^G + NSEC_{tr,t}^G + NSEC_{rv,t}^G \tag{M.181}$$

$$NDEPO_t^G = NDEPO_{t-1}^G + NDEPO_{tr,t}^G + NDEPO_{rv,t}^G \tag{M.182}$$

$$NLOA_t^G = NLOA_{t-1}^G + NLOA_{tr,t}^G + NLOA_{rv,t}^G \tag{M.183}$$

$$NEQ_t^G = NEQ_{t-1}^G + NEQ_{tr,t}^G + NEQ_{rv,t}^G \tag{M.184}$$

$$NINSU_t^G = NINSU_{t-1}^G + NINSU_{tr,t}^G + NINSU_{rv,t}^G \tag{M.185}$$

$$NDERV_t^G = NDERV_{t-1}^G + NDERV_{tr,t}^G + NDERV_{rv,t}^G \quad \text{(M.186)}$$

$$NGOLD_t^G = NGOLD_{t-1}^G + NGOLD_{tr,t}^G + NGOLD_{rv,t}^G \quad \text{(M.187)}$$

$$NCRED_t^G = NCRED_{t-1}^G + NCRED_{tr,t}^G + NCRED_{rv,t}^G \quad \text{(M.188)}$$

$$NINT_t^G = r_t^{DEP} * (NDEPO_t^G + NDERV_t^G + NTCRED_t^G) + r_t^{SEC} * NSEC_t^G + r_t^{LOA} * NLOA_t^G \quad \text{(M.189)}$$

$$NDIV_t^G = r_t^{DIVD} * NEQ_t^G \quad \text{(M.190)}$$

$$NOIR_t^G = r_t^{INSU} * NINSU_t^G \quad \text{(M.191)}$$

Rest of the world

$$YD_t^{ROW} = M_t^{tot} - X_t^{tot} + W_t^{ROW} - (NIMTax_t + NITAX_t^{ROW}) + NINT_t^{ROW} + NDIV_t^{ROW} + NOIR_t^{ROW} + NREFDI_t^{ROW} + NSCON_{r,t}^{ROW} - NSBEN_{p,t}^{ROW} + OCT_t^{ROW} \quad \text{(M.192)}$$

$$NL_t^{ROW} = YD_t^{ROW} - CT_t^{ROW} - NP_t^{ROW} \quad \text{(M.193)}$$

$$FNL_t^{ROW} = NGOLD_{t,tr}^{ROW} + NDEPO_{t,tr}^{ROW} + NSEC_{t,tr}^{ROW} + NLOA_{t,tr}^{ROW} + NEQ_{t,tr}^{ROW} + NINSU_{t,tr}^{ROW} + NDERV_{t,tr}^{ROW} + NTCRED_{t,tr}^{ROW} \quad \text{(M.194)}$$

$$FNW_t^{ROW} = NGOLD_t^{ROW} + NDEPO_t^{ROW} + NSEC_t^{ROW} + NLOA_t^{ROW} + NEQ_t^{ROW} + NINSU_t^{ROW} + NDERV_t^{ROW} + NTCRED_t^{ROW} \quad \text{(M.195)}$$

$$NSEC_{tr,t}^{ROW} = -\left(NSEC_{tr,t}^{NFC} + NSEC_{tr,t}^{H} + NSEC_{tr,t}^{G} + NSEC_{tr,t}^{ROW}\right) \quad \text{(M.196)}$$

$$NTCRED_{tr,t}^{ROW} = -\left(NTCRED_{tr,t}^{NFC} + NTCRED_{tr,t}^{H} + NTCRED_{tr,t}^{G} + NTCRED_{tr,t}^{ROW}\right) \quad \text{(M.197)}$$

$$NDEPO_t^{ROW} = NDEPO_{t-1}^{ROW} + NDEPO_{tr,t}^{ROW} + NDEPO_{rv,t}^{ROW} \quad \text{(M.198)}$$

$$NLOA_t^{ROW} = NLOA_{t-1}^{ROW} + NLOA_{tr,t}^{ROW} + NLOA_{rv,t}^{ROW} \quad \text{(M.199)}$$

$$NEQ_t^{ROW} = NEQ_{t-1}^{ROW} + NEQ_{tr,t}^{ROW} + NEQ_{rv,t}^{ROW} \quad \text{(M.200)}$$

$$NINSU_t^{ROW} = NINSU_{t-1}^{ROW} + NINSU_{tr,t}^{ROW} + NINSU_{rv,t}^{ROW} \quad \text{(M.201)}$$

$$NDERV_t^{ROW} = NDERV_{t-1}^{ROW} + NDERV_{tr,t}^{ROW} + NDERV_{rv,t}^{ROW} \quad \text{(M.202)}$$

$$NGOLD_t^{ROW} = NGOLD_{t-1}^{ROW} + NGOLD_{tr,t}^{ROW} + NGOLD_{rv,t}^{ROW} \quad \text{(M.203)}$$

$$NINT_t^{ROW} = r_t^{DEP} * (NDEPO_t^{ROW} + NDERV_t^{ROW} + NTCRED_t^{ROW}) + r_t^{SEC} * NSEC_t^{ROW} + r_t^{LOA} * NLOA_t^{ROW} \quad \text{(M.204)}$$

$$NDIV_t^{ROW} = r_t^{DIVD} * NEQ_t^{ROW} \quad \text{(M.205)}$$

$$NOIR_t^{ROW} = r_t^{INSU} * NINSU_t^{ROW} \quad \text{(M.206)}$$

Aggregate variables

$$C_t^{agg} = C_{dom,t}^{tot} + M_{C,t}^{tot} + M_{cons,t}^{duty} + C_{ctax,t}^{tot} + C_{VAT,t}^{tot} \quad \text{(M.207)}$$

$$GOV_t^{agg} = GOV_{dom,t}^{tot} + M_{GOV,t}^{tot} + M_{gov,t}^{duty} + GOV_{ctax,t} + GOV_{VAT,t} \quad \text{(M.208)}$$

$$INV_t^{agg} = INV_{dom,t}^{tot} + M_{INV,t}^{tot} + M_{inv,t}^{duty} + INV_{ctax,t} + INV_{VAT,t} \quad \text{(M.209)}$$

$$INVENT_t^{agg} = INVENT_{dom,t}^{tot} + M_{INVENT,t}^{tot} + M_{invent,t}^{duty} + INVENT_{ctax,t} + INVENT_{VAT,t} \quad \text{(M.210)}$$

$$X_t^{agg} = X_{dom,t}^{tot} + M_{X,t}^{tot} + M_{x,t}^{duty} + X_{ctax,t} + X_{VAT,t} \quad \text{(M.211)}$$

$$M_t^{agg} = \sum_{n=1}^{9}\sum_{p=1}^{7} Z_{im,t}^{n\,p} + \sum_{n=1}^{9} Z_{uim,t}^{n} + M_{C,t}^{tot} + M_{INVENT,t}^{tot} + M_{GOV,t}^{tot} + M_{INV,t}^{tot} + M_{X,t}^{tot} \quad \text{(M.212)}$$

$$Y_t = C_t^{agg} + GOV_t^{agg} + INV_t^{agg} + INVENT_t^{agg} + X_t^{agg} - M_t^{agg} \quad \text{(M.213)}$$

$$YF_t = Y_t - (CT_t^{tot} + VAT_t^{tot} + OPT_t^{tot} + M_{duty,t}^{tot}) \quad \text{(M.214)}$$

$$ws_t = \frac{W_t^{NFC}}{YF_t} \quad \text{(M.215)}$$

$$ps_t = 1 - ws \quad \text{(M.216)}$$

III.) Environmental variables

Energy

$$ENERGY_{sup,t}^{n} = D_{sup,t}^{ENERGY,n} * prod_t^n \quad \text{(M.217)}$$

$$ENERGY_{sup,t}^{tot} = \sum_{n=1}^{9} ENERGY_{sup,t}^{n} + ENERGY_{sup,t}^{M} + ENERGY_{sup,t}^{Waste} + ENERGY_{sup,t}^{RE} \quad \text{(M.218)}$$

$$ENERGY_{use,t}^{n} = D_{use,t}^{ENERGY,n} * prod_t^n \quad \text{(M.219)}$$

$$ENERGY_{use,t}^{tot} = \sum_{n=1}^{9} ENERGY_{use,t}^{n} + ENERGY_{use,t}^{HH} + ENERGY_{use,t}^{DL} + ENERGY_{use,t}^{X} \quad \text{(M.220)}$$

$$ENERGY_{use,t}^{HH} = D_{use,t}^{ENERGY,HH} * c_t^{dom} \quad \text{(M.221)}$$

$$Inv_{Delta,t}^{ENERGY} = ENERGY_{sup,t}^{tot} - ENERGY_{use,t}^{tot} \quad \text{(M.222)}$$

$$Inv_t^{ENERGY} = Inv_{t-1}^{ENERGY} + Inv_{Delta,t}^{ENERGY} \quad \text{(M.223)}$$

$$Coil_{res,t}^{gj} = Coil_{res,t-1}^{gj} - \sum_{n=1}^{9} Coil_{sup,t-1}^{n} + Coil_{otc,t-1}^{gj} \quad \text{(M.224)}$$

$$NTgas_{res,t}^{gj} = NTgas_{res,t-1}^{gj} - \sum_{n=1}^{9} NTgas_{extr,sup,t-1}^{n} + NTgas_{otc,t-1}^{gj} \tag{M.225}$$

Emissions

$$EMISSION_{DIRECT,t}^{n} = \sum_{ENERGY=1}^{21} ENERGY_{use,t}^{n} * EMISSION_{coef,t}^{ENERGY,n} \tag{M.226}$$

$$EMISSION_{INDIRECT,t}^{n} = EMISSION_{coef,t}^{INDIRECT,n} * prod_{t}^{n} \tag{M.227}$$

$$EMISSION_{tot,t}^{n} = EMISSION_{INDIRECT,t}^{n} + EMISSION_{DIRECT,t}^{n} \tag{M.228}$$

$$EMISSION_{DIRECT,t}^{HH} = \sum_{ENERGY=1}^{21} ENERGY_{use,t}^{HH} * EMISSION_{coef,t}^{ENERGY,HH} \tag{M.229}$$

$$EMISSION_{INDIRECT,t}^{HH} = EMISSION_{coef,t}^{INDIRECT,HH} * c_{t}^{dom} \tag{M.230}$$

$$EMISSION_{tot,t}^{HH} = EMISSION_{DIRECT,t}^{HH} + EMISSION_{INDIRECT,t}^{HH} \tag{M.231}$$

$$EMISSION_{t}^{tot} = \sum_{n=1}^{9} EMISSION_{tot,t}^{n} + EMISSION_{tot,t}^{HH} \tag{M.232}$$

$$CO2E_{t}^{n} = CO2_{t}^{n} + SF6_{t}^{n} + PFC_{t}^{n} + HFC_{t}^{n} + 25 * CH4_{t}^{n} + 298 * N2O_{t}^{n} \tag{M.233}$$

$$CO2E_{t}^{tot} = CO2_{t}^{tot} + SF6_{t}^{tot} + PFC_{t}^{tot} + HFC_{t}^{tot} + 25 * CH4_{t}^{tot} + 298 * N2O_{t}^{tot} \tag{M.234}$$

Environmental taxes

$$ENV_{tax,t}^{n} = CO2E_{rate,t}^{n} * CO2E_{t}^{n} \tag{M.235}$$

8.7. Variables & Parameters

Industry level variables	***notation***
Total production, costs and profits	
Nominal production for domestic industries	$PROD_{dom,t}^{n}$
Real production for domestic industries	$prod_{dom,t}^{n}$
Nominal cost for domestic industries	$COST_{dom,t}^{n}$
Nominal profits for domestic industries	$PROFIT_{t}^{n}$
Inputs in production, and taxes	
Real total inputs in domestic production industry $\boldsymbol{n}$ and produced by either domestic or foreign industry $\boldsymbol{i}$.	$z_{t}^{i\ n}$
Real inputs in domestic production industry $\boldsymbol{n}$ and produced by domestic industry $\boldsymbol{i}$.	$z_{dom,t}^{i\ n}$
Real inputs in domestic production industry $\boldsymbol{n}$ and produced by foreign industry $\boldsymbol{i}$.	$z_{im,t}^{i\ n}$

Nominal inputs in domestic production industry $\boldsymbol{n}$ and produced by domestic industry $\boldsymbol{i}$.	$Z_{dom,t}^{i\,n}$
Nominal inputs in domestic production industry $\boldsymbol{n}$ and produced by foreign industry $\boldsymbol{i}$.	$Z_{im,t}^{i\,n}$
Real inputs in domestic production industry $\boldsymbol{n}$ produced by an unspecified industry	$z_{uim,t}^{n}$
Nominal inputs in domestic production industry $\boldsymbol{n}$ produced by an unspecified industry	$Z_{uim,t}^{n}$
Commodity taxes paid by industry n	CT_t^n
Commodity taxes paid through final consumption by households	$C_{ctax,t}^{tot}$
Commodity taxes paid through final consumption by government	$GOV_{ctax,t}^{tot}$
Commodity taxes paid through spending on investment products	$INV_{ctax,t}^{tot}$
Commodity taxes paid through spending on inventories.	$INVENT_{ctax,t}^{tot}$
Commodity taxes paid through exports	$X_{ctax,t}^{tot}$
Value added taxes paid by industry n	VAT_t^n
Value added taxes paid through final consumption by households.	$C_{VAT,t}^{tot}$
Value added taxes paid through final consumption by government.	$GOV_{VAT,t}^{tot}$
Value added taxes paid through spending on investment products.	$INV_{VAT,t}^{tot}$
Value added taxes paid through spending on inventories.	$INVENT_{VAT,t}^{tot}$
Value added taxes paid through exports.	$X_{VAT,t}^{tot}$
Other production taxes paid by industry n	$OTP_{tax,t}^{n}$
Non-environmental other production taxes paid by industry n	$NE_{tax,t}^{n}$
Import duties paid by industry n	$M_{duty,t}^{n}$
Import duties paid through final consumption by households.	$M_{CONS,t}^{duty}$
Import duties paid through final consumption by government.	$M_{GOV,t}^{duty}$
Import duties paid through spending on investment products.	$M_{INV,t}^{duty}$
Import duties paid through spending on inventories.	$M_{INVENT,t}^{duty}$
Import duties paid through exports.	$M_{X,t}^{duty}$
Total nominal imports associated with final consumption	M_c^{tot}
Total nominal imports associated with investments	M_{inv}^{tot}
Total nominal imports associated with changes in inventories.	M_{invent}^{tot}
Total nominal imports associated with government spending	M_{gov}^{tot}
Total nominal imports associated with exports	M_{ex}^{tot}
Consumption	
Total real consumption	c_t^{tot}
Total nominal consumption	C_t^{tot}
Real consumption in product type p	c_t^p
Nominal consumption in product type p	C_t^p
Real consumption share for industry specific products	$\gamma_t^{c^{spec}}$
Real consumption share for bread products	$\gamma_t^{c^{110}}$
Real consumption share for meat products	$\gamma_t^{c^{120}}$
Real consumption share for fish products	$\gamma_t^{c^{130}}$
Real consumption share for dairy products	$\gamma_t^{c^{140}}$
Real consumption share for fruit and vegetables products	$\gamma_t^{c^{160}}$
Real consumption share for other food products	$\gamma_t^{c^{180}}$
Total real consumption of type p that is domestically produced	$c_{dom,t}^{tot,p}$
Total real consumption of type p that is imported	$c_{im,t}^{tot,p}$
Total nominal consumption of type p that is domestically produced	$C_{dom,t}^{tot,p}$
Total nominal consumption of type p that is imported	$C_{im,t}^{tot,p}$
Import share for product type p	$\phi_{c,t}^p$
Real consumption of product type p produced by domestic industry n.	$c_{dom,t}^{p,n}$
Real consumption of product type p produced by foreign industry n.	$c_{im,t}^{n\,p}$
Real unspecified imports associated with final consumption of product type p.	$c_{uim,t}^{p}$

Nominal consumption of product type p produced by domestic industry n.	$C_{dom,t}^{n\,p}$
Nominal consumption of product type p produced by foreign industry n.	$C_{im,t}^{n\,p}$
Nominal unspecified imports associated with final consumption of product type p.	$C_{uim,t}^{p}$
Real consumption in industry n	$c_{dom,t}^{tot,n}$
Real consumption in foreign industry n	$c_{im,t}^{tot,n}$
Nominal consumption in industry n	$C_{dom,t}^{tot,n}$
Nominal consumption in foreign industry n	$C_{im,t}^{tot,n}$
Total real consumption of domestically produced products	$c_{dom,t}^{tot}$
Total real consumption of foreign produced products	$c_{im,t}^{tot}$
Total nominal consumption of domestically produced products	$C_{dom,t}^{tot}$
Total nominal consumption of foreign produced products	$C_{im,t}^{tot}$
Investment	
Total real investments	inv_t^{tot}
Total nominal investments	INV_t^{tot}
Total real investment products bought from industry n.	inv_t^{n}
Total Nominal investment in products bought from industry n.	INV_t^{n}
Real investment supplied by domestic industry n.	$inv_{dom,t}^{n}$
Real investment supplied by foreign industry n.	$inv_{im,t}^{n}$
Real investment supplied though unspecified imports.	$inv_{uim,t}$
Total real investments supplied by domestic industries.	$inv_{dom,t}^{tot}$
Total real investments supplied by foreign industries.	$inv_{im,t}^{tot}$
Total nominal investments supplied by domestic industries.	$INV_{dom,t}^{tot}$
Total nominal investments supplied by foreign industries.	$INV_{im,t}^{tot}$
Nominal investment supplied though unspecified imports.	$INV_{uim,t}$
Exports	
Total real exports	x_t^{tot}
Total nominal exports	X_t^{tot}
World real imports of industry n products	m_t^{n*}
Real exports for industry n.	x_t^{n}
Nominal exports for industry n.	X_t^{n}
Real exports of domestically produced products for industry n.	$x_{dom,t}^{n}$
Real exports of foreign produced products for industry n.	$x_{im,t}^{n}$
Real exports supplied though unspecified imports.	$x_{uim,t}$
Nominal exports of domestically produced products for industry n.	$X_{dom,t}^{n}$
Nominal exports of foreign produced products for industry n.	$X_{im,t}^{n}$
Nominal exports supplied though unspecified imports.	$X_{uim,t}$
Total real exports of domestically produced products.	$x_{dom,t}^{tot}$
Total real exports of foreign produced products.	$x_{im,t}^{tot}$
Total nominal exports of domestically produced products.	$X_{dom,t}^{tot}$
Total nominal exports of foreign produced products.	$X_{im,t}^{tot}$
Government spending (exogenous in baseline)	
Total real Government spending	gov_t^{tot}
Total nominal Government spending	GOV_t^{tot}
Real Government spending for industry n.	gov_t^{n}
Nominal Government spending for industry n.	GOV_t^{n}
Real government spending of domestically produced products for industry n.	$gov_{dom,t}^{n}$
Real government spending of foreign produced products for industry n.	$gov_{im,t}^{n}$
Real government spending associated with unspecified imports.	$gov_{uim,t}$
Nominal Government spending of domestically produced products for industry n.	$GOV_{dom,t}^{n}$
Nominal government spending of foreign produced products for industry n.	$GOV_{im,t}^{n}$
Nominal government spending associated with unspecified imports.	$GOV_{uim,t}$
Total real government spending of domestically produced products.	$gov_{dom,t}^{tot}$

Total real government spending for foreign produced products.	$gov_{im,t}^{tot}$
Total nominal government spending of domestically produced products.	$GOV_{dom,t}^{tot}$
Total nominal government spending for foreign produced products.	$GOV_{im,t}^{tot}$
Inventories (exogenous in baseline)	
Total real change in inventories	$invent_t^{tot}$
Total nominal change in inventories	$INVENT_t^{tot}$
Real change in inventories for industry n.	$invent_t^n$
Nominal change in inventories for industry n.	$INVENT_t^n$
Real change in inventories of domestically produced products for industry n.	$invent_{dom,t}^n$
Real change in inventories of foreign produced products for industry n.	$invent_{im,t}^n$
Real change in inventories associated with unspecified imports.	$invent_{uim,t}$
Nominal change in inventories of domestically produced products for industry n.	$INVENT_{dom,t}^n$
Nominal change in inventories of foreign produced products for industry n.	$INVENT_{im,t}^n$
Nominal change in inventories associated with unspecified imports.	$INVENT_{uim,t}$
Total real change in inventories of domestically produced products.	$invent_{dom,t}^{tot}$
Total real change in inventories for foreign produced products.	$invent_{im,t}^{tot}$
Total nominal change in inventories of domestically produced products.	$INVENT_{dom,t}^{tot}$
Total nominal change in inventories for foreign produced products.	$INVENT_{im,t}^{tot}$
Aggregate Final demand	
Aggregate nominal final consumption	C_t^{agg}
Aggregate nominal government spending	GOV_t^{agg}
Aggregate nominal investments	INV_t^{agg}
Aggregate nominal change in inventories	$INVENT_t^{agg}$
Aggregate nominal exports	X_t^{agg}
Aggregate nominal imports	M_t^{agg}
Aggregate nominal gross domestic product	Y_t
Labor market	
Employment in industry n.	EMP_t^n
Total employment	EMP_t^{tot}
Total unemployment	$UNEMP_t$
Unemployment rate	UR_t
Wage bill for industry n.	W_t^n
Wage rate in industry n.	$Wage_t^n$
General wage	$Wage_t^{gen}$
Targeted general wage	$Wage_t^{gen,T}$
Productivity in industry n.	a_t^n
Price indices and real exchange rate	
Producer price index for industry n.	py_t^n
Producer price index for foreign industries n.	pm_t^n
Producer price index for unspecified imports.	pm_t^{un}
Real exchange rate for industry n.	rer_t^n
Nominal exchange rate	xr_t
Tax adjusted consumer price index for product type p.	$ppcon_t^{p,tax}$
Tax-rate for product type p.	$tax_{rate,t}^p$
Consumer price index for domestically produced products of product type p.	$ppcon_{dom,t}^p$
Consumer price index for foreign produced products of product type p.	$ppcon_{im,t}^p$
Real exchange rate for product type p.	rer_t^p

Note: The notation for industries (n) covers the following industries: Agricultural, Forestry, Fishery, Mining, Manufacturing of food, Energy supply and refineries, Other energy intensive industries, Financial corporations, Other industries.

The notation for product types (p) covers the following products: bread (c_t^{110}), meat (c_t^{120}), fish (c_t^{130}), dairy (c_t^{140}), fruits and vegetables (c_t^{160}), other food products (c_t^{180}), and finally industry specific products (c_t^{spec}).

Sector level variables	***notation***
Sectoral gross operation surplus	$B2^{NFC}, B2^{H}, B2^{FC}, B2^{G}$
Sectoral nominal investments	$I_t^{NFC}, I_t^{H}, I_t^{FC}, I_t^{G}$
Sectoral nominal change in inventories.	$INVENT_t^{NFC}, INVENT_t^{H}, INVENT_t^{FC}, INVENT_t^{G}$
Sectoral nominal income received.	$Y_t^{NFC}, Y_t^{H}, Y_t^{FC}, Y_t^{G}$
Sectoral net income received from interest bearing assets.	$NINT_t^{NFC}, NINT_t^{H}, NINT_t^{FC}, NINT_t^{G}, NINT_t^{ROW}$
Sectoral net income received from dividends.	$NDIV_t^{NFC}, NDIV_t^{H}, NDIV_t^{FC}, NDIV_t^{G}, NDIV_t^{ROW}$
Sectoral net income received from other investments.	$NOIR_t^{NFC}, NOIR_t^{H}, NOIR_t^{FC}, NOIR_t^{G}, NOIR_t^{ROW}$
Sectoral net income received from retained earnings on FDI.	$NREFDI_t^{NFC}, NREFDI_t^{H}, NREFDI_t^{FC}, NREFDI_t^{G}, NREFDI_t^{ROW}$
Sectoral net other current transfers	$OCT_t^{NFC}, OCT_t^{H}, OCT_t^{FC}, OCT_t^{G}, OCT_t^{ROW}$
Sectoral disposable income.	$YD_t^{NFC}, YD_t^{H}, YD_t^{FC}, YD_t^{G}, YD_t^{ROW}$
Sectoral savings.	$S_t^{NFC}, S_t^{H}, S_t^{FC}, S_t^{G}, S_t^{ROW}$
Sectoral net lending	$NL_t^{NFC}, NL_t^{H}, NL_t^{FC}, NL_t^{G}, NL_t^{ROW}$
Sectoral acquisition less disposals of non-produced non-financial assets.	$NP_t^{NFC}, NP_t^{H}, NP_t^{FC}, NP_t^{G}, NP_t^{ROW}$
Sectoral capital transfers	$CT_t^{NFC}, CT_t^{H}, CT_t^{FC}, CT_t^{G}, CT_t^{ROW}$
Sectoral financial net lending.	$FNL_t^{NFC}, FNL_t^{H}, FNL_t^{FC}, FNL_t^{G}, FNL_t^{ROW}$
Sectoral net transactions of deposits.	$NDEPO_{tr,t}^{NFC}, NDEPO_{tr,t}^{H}, NDEPO_{tr,t}^{FC}, NDEPO_{tr,t}^{G}, NDEPO_{tr,t}^{ROW}$
Sectoral net transactions of securities.	$NSEC_{tr,t}^{NFC}, NSEC_{tr,t}^{H}, NSEC_{tr,t}^{FC}, NSEC_{tr,t}^{G}, NSEC_{tr,t}^{ROW}$
Sectoral net transactions of Loans.	$NLOA_{tr,t}^{NFC}, NLOA_{tr,t}^{H}, NLOA_{tr,t}^{FC}, NLOA_{tr,t}^{G}, NLOA_{tr,t}^{ROW}$
Sectoral net transactions of equities.	$NEQ_{tr,t}^{NFC}, NEQ_{tr,t}^{H}, NEQ_{tr,t}^{FC}, NEQ_{tr,t}^{G}, NEQ_{tr,t}^{ROW}$
Sectoral net transactions of insurance/tech. reserves.	$NINSU_{tr,t}^{NFC}, NINSU_{tr,t}^{H}, NINSU_{tr,t}^{FC}, NINSU_{tr,t}^{G}, NINSU_{tr,t}^{ROW}$
Sectoral net transactions of financial derivatives.	$NDERV_{tr,t}^{NFC}, NDERV_{tr,t}^{H}, NDERV_{tr,t}^{FC}, NDERV_{tr,t}^{G}, NDERV_{tr,t}^{ROW}$
Sectoral net transactions of trade credits.	$NTCRED_{tr,t}^{NFC}, NTCRED_{tr,t}^{H}, NTCRED_{tr,t}^{FC}, NTCRED_{tr,t}^{G}, NTCRED_{tr,t}^{ROW}$
Sectoral net transactions of gold.	$NGOLD_{t,tr}^{FC}, NGOLD_{t,tr}^{ROW}$
Sectoral financial net wealth.	$FNW_t^{NFC}, FNW_t^{H}, FNW_t^{FC}, FNW_t^{G}, FNW_t^{ROW}$
Sectoral stock of net deposits.	$NDEPO_t^{NFC}, NDEPO_t^{H}, NDEPO_t^{FC}, NDEPO_t^{G}, NDEPO_t^{ROW}$
Sectoral stock of net securities.	$NSEC_t^{NFC}, NSEC_t^{H}, NSEC_t^{FC}, NSEC_t^{G}, NSEC_t^{ROW}$
Sectoral stock of net loans.	$NLOA_t^{NFC}, NLOA_t^{H}, NLOA_t^{FC}, NLOA_t^{G}, NLOA_t^{ROW}$
Sectoral stock of net equities.	$NEQ_t^{NFC}, NEQ_t^{H}, NEQ_t^{FC}, NEQ_t^{G}, NEQ_t^{ROW}$
Sectoral stock of net insurance/tech. reserves.	$NINSU_t^{NFC}, NINSU_t^{H}, NINSU_t^{FC}, NINSU_t^{G}, NINSU_t^{ROW}$
Sectoral stock of net financial derivatives.	$NDERV_t^{NFC}, NDERV_t^{H}, NDERV_t^{FC}, NDERV_t^{G}, NDERV_t^{ROW}$
Sectoral stock of net trade credits.	$NTCRED_t^{NFC}, NTCRED_t^{H}, NTCRED_t^{FC}, NTCRED_t^{G}, NTCRED_t^{ROW}$
Sectoral stock of net gold.	$NGOLD_t^{FC}, NGOLD_t^{ROW}$
Loans to capital ratio for non-financial corporations	$loa_{ratio,t}^{NFC}$
Sectoral net lending adjustment term.	$NL_{adj,t}^{NFC}, NL_{adj,t}^{H}, NL_{adj,t}^{FC}, NL_{adj,t}^{G}, NL_{adj,t}^{ROW}$
Total wage bill paid by non-financial corporations.	$W_{p,t}^{NFC}$
Total wage bill received by domestic households	$W_{r,t}^{H}$
Net wage bill received by foreign workers.	W_t^{ROW}
Number of people unemployed	$UNEMP_t$
Number of people in the labor force	LF_t
Unemployment rate	UR_t
Sectoral social contributions.	$SCON_t^{H}, SCON_{t,p}^{FC}, SCON_{t,p}^{G}, SCON_{t,p}^{ROW}$
Sectoral social benefits.	$SBEN_{t,r}^{H}, SBEN_{t,r}^{FC}, SBEN_{t,r}^{G}, SBEN_{t,r}^{ROW}$
Sectoral income tax	$ITAX_t^{NFC}, ITAX_t^{H}, ITAX_t^{FC}, ITAX_t^{G}, ITAX_t^{ROW}$
Adjustment term for household's pension.	PEN_r^{adj}
Households share of net equities held to total financial wealth	$EQ_{ratio,t}^{H}$
Households share of loans to disposable income.	$LOA_{ratio,t}^{H}$
Total net production taxes	$NTAX_t^{prod}$
Total net other production taxes	OPT_t^{prod}
Total net import taxes	$NIMTax_t$
Import taxes received by Denmark	$IMTAX_{r,t}$

Import taxes paid by Denmark	$IMTAX_{p,t}$
Other net import taxes paid by Denmark.	$OIMTAX_{p,t}$
Interest rate on securities	r_t^{SEC}
Interest rate on deposits	r_t^{DEPO}
Interest rate on loans	r_t^{LOA}
Interest rate on dividends	r_t^{DIVD}
Interest rate on pensions and insurance	r_t^{INSU}
Sectoral real capital stock	$k_t^{NFC}, k_t^{H}, k_t^{FC}, k_t^{G}$
Sectoral nominal capital stock	$K_t^{NFC}, K_t^{H}, K_t^{FC}, K_t^{G}$
capacity utilization rate	u_t

Environmental variables	***notation***
Energy supply in industry n.	$ENERGY_{sup,t}^{n}$
Energy supply imported.	$ENERGY_{sup,t}^{M}$
Energy supply in the form of waste.	$ENERGY_{sup,t}^{Waste}$
Energy supply in the form of renewable energy.	$ENERGY_{sup,t}^{RE}$
Total energy supply	$ENERGY_{sup,t}^{tot}$
Energy usage in industry n.	$ENERGY_{use,t}^{n}$
Energy exported for usage outside Denmark	$ENERGY_{use,t}^{X}$
Distribution losses related to energy usage.	$ENERGY_{use,t}^{DL}$
Energy usage by households.	$ENERGY_{use,t}^{HH}$
Change in inventories for energy types.	$Inv_{Delta,t}^{ENERGY}$
Inventory stock of energy.	Inv_{t}^{ENERGY}
New discoveries of crude oil reserves.	$Coil_{otc,t-1}^{gj}$
Stock of crude oil reserve.	$Coil_{res,t}^{gj}$
New discoveries of natural gas for extraction reserves.	$NTgas_{otc,t-1}^{gj}$
Stock of natural gas for extraction reserve.	$NTgas_{res,t}^{gj}$
Emissions	
Emission for industry n directly related to energy.	$EMISSION_{DIRECT,t}^{n}$
Emission for industry n unrelated to energy.	$EMISSION_{INDIRECT,t}^{n}$
Total emissions for industry n (both direct and indirect)	$EMISSION_{tot,t}^{n}$
Emissions for households directly related to energy.	$EMISSION_{DIRECT,t}^{HH}$
Emissions for households unrelated to energy.	$EMISSION_{INDIRECT,t}^{HH}$
Total emissions for households.	$EMISSION_{tot,t}^{HH}$
Total emissions in the Danish economy	$EMISSION_{t}^{tot}$
CO2-equivelant emissions for each industry n.	$CO2E_{t}^{n}$
Total CO2-equivelant emissions in the Danish economy.	$CO2E_{t}^{tot}$

Note: The notation for energy ($ENERGY$) covers the 21 types of energy (Coil (crude oil), Oilp: Oil products), RefG (Refinery gas), GasT (Gasoline for transportation), FGas (jet fuel), FGasBunk (Jet fuel bunkered), DieT (Diesel for transportation), DietTBunk (Diesel for transportation - bunkered), NGasExt (Natural gas extraction), NGasCons (Natural gas consumption – incl. city gas), CC (coal and smoke), Waste (Waste), RE (Renewable energy), Straw (Straw), FW (Firewood and wood chips), WP (wood pellets), BioG (Bio gas), BBB (Biodiesel, bioethanol and bio oil), El (electricity), DHeat (District heat).).

The notation for emissions ($EMISSION$) covers the 6 types of emissions (CO2 (carbon dioxide), N2O (nitrous oxide), CH4 (methane), SF6 (sulfur hexafluoride), PFC (perfluorocarbons), and HFC (Hydrofluorocarbons))

The notation for industries (n) covers the following industries: Agricultural, Forestry, Fishery, Mining, Manufacturing of food, Energy supply and refineries, Other energy intensive industries, Financial corporations, Other industries.

Parameters	*Notation*
Economic parameters	
Technical coefficient relating output for industry *n* to inputs bought by industry *n* in industry *i*.	$a_t^{i\,n}$
Import share for industry n.	$\phi_{z,t}^{i\,n}$
Import share of investment products for industry n.	$\phi_{INV,t}^{n}$
Import share of exports for industry n.	$\phi_{X,t}^{n}$
Intercept in the equation for import shares for product type p in final consumption.	$\beta_0^{c^p}$
Intercept in the equation for import shares for product type p for industries.	$\beta_0^{z^{i\,n}}$
Import elasticity for final consumption in product type p.	$\beta_1^{c^p}$
Import elasticity for inputs produced by industry n.	$\beta_1^{z^n}$
Wage premium for industry n.	ω_0^n
Wage bargaining parameter for industry n.	ω_1^n
Share of product p supplied by domestic industry n for final consumption	$\lambda_{dom,t}^{n\,p}$
Share of product p supplied by foreign industry n for final consumption	$\lambda_{im,t}^{n\,p}$
Share of investment products bought from industry n.	$\lambda_{inv,t}^{n}$
Share of unspecified imports to total production in industry n.	$\gamma_{uim,t}^{n}$
Share of unspecified imports to total production for final good p.	$\gamma_{uim,t}^{p}$
Share of unspecified imports to total investments.	$\gamma_{uim,t}^{inv}$
Parameter set to match first observation in consumption share $\gamma_t^{c^p}$.	θ^p
Rate of substitution for consumption in nest 1.	σ^{nest1}
Rate of substitution for consumption in nest 2	σ^{nest2}
Intercept in export equation. Set to match starting value of $\frac{x_t^n}{m_t^{n*}}$ for industry n.	α_0^n
Export elasticity of industry n.	α_1^n
Interest rate on deposits	r_t^{DEP}
Interest rate on securities	r_t^{SEC}
Interest rate on loans	r_t^{LOA}
Interest rate on dividends	r_t^{DIVD}
Interest rate on insurance, pension, and standardized guarantee schemes.	r_t^{INSU}
Price index for import in industry n.	pm_t^n
Price index for unspecified imports.	$pm_{uim,t}$
Mark-up rate for industry n.	μ_t^n
Share of industry *n* gross operating surplus and mixed income associated with the household sector.	$s_{H,t}^{profit,n}$
Share of industry *n* gross operating surplus and mixed income associated with the financial corporation sector.	$s_{FC,t}^{profit,n}$
Share of industry *n* gross operating surplus and mixed income associated with the government sector.	$s_{G,t}^{profit,n}$

Share distributing total investments to the household sector.	$S_{H,t}^{inv}$
Share distributing total investments to the financial corporation sector.	$S_{FC,t}^{inv}$
Share distributing total investments to the government sector.	$S_{G,t}^{inv}$
Adjustment term used in import equation for industries	$adj_{\phi,t}^{n}$
Adjustment term used for import equation for final consumption products.	$adj_{\phi,t}^{p}$
Adjustment term used in export equation for industries.	$adj_{x,t}^{n}$
Adjustment term used in the transition of profits from industry to sectoral level.	$\mathrm{adj}_t^{\mathrm{H}}$
Adjustment term used in the transition of profits from industry to sectoral level	$\mathrm{adj}_t^{\mathrm{FC}}$
Adjustment term used in the transition of profits from industry to sectoral level	$\mathrm{adj}_t^{\mathrm{G}}$
Environmental parameters	
Energy supply coefficient for industry n.	$D_{sup,t}^{ENERGY,n}$
Energy usage coefficient for industry n.	$D_{use,t}^{ENERGY,n}$
Energy usage coefficient for households.	$D_{use,t}^{ENERGY,HH}$
Emission coefficient for emissions directly from energy usage in industry n.	$EMISSION_{coef,t}^{ENERGY,n}$
Emission coefficient for emissions not directly from energy usage in industry n.	$EMISSION_{coef,t}^{INDIRECT,n}$
Emission coefficient for emissions directly from energy usage by households.	$EMISSION_{coef,t}^{ENERGY,HH}$
Emission coefficient for emissions not directly from energy usage by the households.	$EMISSION_{coef,t}^{INDIRECT,HH}$
CO2-equivelant tax rate for each industry n.	$CO2E_{rate,t}^{n}$

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