Facit til kursusgang 25: Bestemte integraler 1

1. Svarene er:

$$\frac{7}{3}, \qquad \frac{1}{2}, \qquad \ln(2), \qquad 0.$$

2. Svarene er:

$$\frac{34}{3}, \qquad \frac{26}{3}, \qquad 20+4\ln(2).$$

3. Arealet er $e^3 - e$.

4. Vi har at

$$\int_a^b 1\,dx = [x]_a^b = b - a.$$

5. Da $\sqrt{x} > x^2$ i det givne interval har vi

$$\int_0^1 \sqrt{x} - x^2\,dx = [\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3]_0^1 = \frac{1}{3}.$$

6. Arealet af rektanglet kan beskrives som

$$\int_0^a b\,dx = [bx]_0^a = ba.$$

7. Vi har at

$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = F(c) - F(a) + F(b) - F(c) = F(b) - F(a) = \int_a^b f(x)\,dx.$$

8. Husk at

$$f(x) = \begin{cases} -x, & \text{hvis } x < 0 \\ x, & \text{hvis } x \geq 0. \end{cases}$$

   Anvender vi hintet får fi at

$$\int_{-2}^1 |x|\,dx = \int_{-2}^0 -x\,dx + \int_0^1 x\,dx = [-\frac{1}{2}x^2]_{-2}^0 + [\frac{1}{2}x^2]_0^1 = \frac{5}{2}.$$

9. Vi har at

$$-\int_b^a f(x)\,dx = -(F(a) - F(b)) = F(b) - F(a) = \int_a^b f(x)\,dx.$$

10. Svarene er:

$$\frac{\sqrt{2}}{2}.$$

11. Et muligt svar er $a = \pi$.

12. Arealet er 2.

EKSTRAOPGAVER:

13. Vi har at

$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} x^{\frac{2}{3}}\,dx + \int_{0}^{1} x^{\frac{1}{3}}\,dx = [\frac{3}{5}x^{\frac{5}{3}}]_{-1}^{0} + [\frac{3}{4}x^{\frac{4}{3}}]_{0}^{1} = \frac{27}{20}.$$

14. De to funktioner skærer hinanden når $x = \pm\frac{\sqrt{2}}{2}$. Vi har dermed at

$$\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} 1 - x^2 - x^2\,dx = [x - \frac{2}{3}x^3]_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} = \frac{2\sqrt{2}}{3}.$$

15. Først ser vi at de to tangenter skærer hinanden i $(\frac{5}{4}, 0)$ samt at $y = -4x + 5$ skærer parablen i $(0, 5)$ og $y = x + \frac{5}{4}$ skærer parablen i $(\frac{5}{2}, \frac{5}{2})$. Ved at integrere får vi at

$$\int_{0}^{\frac{5}{4}} x^2 - 4x + 5 - (-4x + 5)\,dx = \int_{0}^{\frac{5}{4}} x^2\,dx = [\frac{1}{3}x^3]_{0}^{\frac{5}{4}} = \frac{125}{192}$$

og at

$$\begin{aligned}\int_{\frac{5}{4}}^{\frac{5}{2}} x^2 - 4x + 5 - (x - \frac{5}{4})\,dx &= \int_{\frac{5}{4}}^{\frac{5}{2}} x^2 - 5x + \frac{25}{4}\,dx \\ &= [\frac{1}{3}x^3 - \frac{5}{2}x^2 + \frac{25}{4}x]_{\frac{5}{4}}^{\frac{5}{2}} \\ &= \frac{125}{24} - \frac{125}{8} + \frac{125}{8} - (\frac{125}{192} - \frac{125}{32} + \frac{125}{16}) \\ &= \frac{125}{24} - \frac{125}{192} - \frac{125}{32} = \frac{1000 - 125 - 750}{192} = \frac{125}{192}.\end{aligned}$$

Dermed ses at de to arealer er lige store. Det totale areal er $\frac{125}{96}$.